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A CHARACTERIZATION OF TWO-DIMENSIONAL RATIONAL SINGULARITIES VIA CORE OF IDEALS

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Abstract. The notion of p_g-ideals for normal surface singularities has been proved to be very useful. On the other hand, the core of ideals has been proved to be very important concept and also very mysterious one. However, the computation of the core of an ideal seems to be given only for very special cases. In this paper, we will give an explicit description of the core of p_g-ideals of normal surface singularities. As a consequence, we give a characterization of rational singularities using the inclusion of the core of integrally closed ideals.

1. Introduction

Let $(A, \mathfrak{m})$ be a two-dimensional excellent normal local domain containing an algebraically closed field. We always assume that $(A, \mathfrak{m})$ is not regular. When $(A, \mathfrak{m})$ is a rational singularity, Lipman [14] proved that any integrally closed $\mathfrak{m}$-primary ideal I is stable, namely, $I^2 = QI$ for some (every) minimal reduction Q, and that if I and I' are integrally closed $\mathfrak{m}$-primary ideals, then the product II' is also integrally closed. (Later, Cutkosky [4] showed that this property characterizes the rational singularities for two-dimensional excellent normal local domains.) These facts play very important role to study ideal theory on a two-dimensional rational singularity.

In [15], the authors introduced the notion of p_g-ideals for two-dimensional excellent normal local domain containing an algebraically closed field and proved that the p_g-ideals inherit nice properties of integrally closed ideals of rational singularities (in a rational singularity, every integrally closed ideal is a p_g-ideal by our definition). Namely, any p_g-ideal I is stable and if I and I' are p_g-ideals, then II' is integrally closed and also a p_g-ideal.

Let $f : X \to \operatorname{Spec} A$ be a resolution of singularity. Then $p_g(A) := \ell_A(H^1(X, \mathcal{O}_X))$ is independent of the choice of a resolution and an important invariant of A (here

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