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Uniform annihilators of local cohomology of extended Rees rings

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Abstract

The purpose of the paper is to study the annihilators of local cohomology of the extended Rees rings. We will prove that if $(A, \mathfrak{m})$ is a noetherian local ring with infinite residue field, and a is a uniform local cohomological annihilator of A, then there is an integer n such that $(at)^n$ is a uniform local cohomological annihilator of the extended Rees ring $A[t, t^{-1}I]$ for any $\mathfrak{m}$-primary ideal I of A.

1 Introduction

Let A be a noetherian ring, I an ideal of A, and t an indeterminate over A. Consider $A[t, t^{-1}]$ as a graded ring in the natural way with $\deg(t) = 1$. The extended Rees ring of A associated to I is the graded subring $R = A[t, t^{-1}I]$ of the ring $A[t, t^{-1}]$. If $I = (a_1, a_2, \cdots, a_r)$ then $R[t, t^{-1}I] = R[t, t^{-1}a_1, \cdots, t^{-1}a_r]$, so that R is also a noetherian ring. It is known that $\dim(R) = \dim(A) + 1$ [cf. Ma].

Let $\mathrm{gr}_I(A) = \bigoplus_{n\geq 0} I^n/I^{n+1}$ be the associated graded ring of A with respect to I. One can show easily that

$$R/tR \cong \mathrm{gr}_I(A) \quad \text{and} \quad R/(1-t)R \cong A,$$

so that we can regard $\mathrm{gr}_I(A)$ as a deformation of the original ring A, with R as total space of the deformation, in the sense that the values $t = 1$ and 0 correspond to A and $\mathrm{gr}_I(A)$, respectively. Such deformation plays an important role in the theory of algebraic geometry [cf. Fu].

It is well known that if $(A, \mathfrak{m})$ is a d-dimensional Cohen-Macaulay (abbr. CM) local ring and I is an ideal generated by a system of parameters $a_1, a_2, \cdots, a_d$, then the extended Rees ring R is also a CM ring [cf, BH], and the result is not valid for all $\mathfrak{m}$-primary ideals. This means that there is a $\mathfrak{m}$-primary ideal I and a maximal ideal Q of $A[t, t^{-1}I]$ such that $H^i_Q(A[t, t^{-1}I])$ may be nonzero for some i ($i < \mathrm{ht}(Q)$). In fact, such local cohomology modules are rarely finitely generated. In this paper, we will study the annihilators of these modules.

Recalling that an element $a \in A$ is said to be a uniform local cohomological annihilator of A, if it is not lying in any minimal prime ideal of A, and for any maximal ideal