Quasi-projective Brauer characters

Yanjun Liu, Wolfgang Willems

PII: S0021-8693(17)30674-9
DOI: https://doi.org/10.1016/j.jalgebra.2017.12.016
Reference: YJABR 16504

To appear in: *Journal of Algebra*

Received date: 25 September 2017

Please cite this article in press as: Y. Liu, W. Willems, Quasi-projective Brauer characters, *J. Algebra* (2018), https://doi.org/10.1016/j.jalgebra.2017.12.016

QUASI-PROJECTIVE BRAUER CHARACTERS

YANJUN LIU
COLLEGE OF MATHEMATICS AND INFORMATION SCIENCE
JIANGXI NORMAL UNIVERSITY, NANCHANG, CHINA

AND

WOLFGANG WILLEMS
FAKULTÄT FÜR MATHEMATIK
OTTO-VON-GUERICKE UNIVERSITÄT, MAGDEBURG, GERMANY
AND
DEPARTAMENTO DE MATEMÁTICAS
UNIVERSIDAD DEL NORTE, BARRANQUILLA, COLOMBIA

Dedicated to Professor B. Huppert on the occasion of his 90th birthday.

Abstract We study p-Brauer characters of a finite group G which are restrictions of generalized characters vanishing on p-singular elements for a fixed prime p dividing the order of G. Such Brauer characters are called quasi-projective. We show that for each irreducible Brauer character φ there exists a minimal p-power, say $p^{a(\varphi)}$, such that $p^{a(\varphi)}\varphi$ is quasi-projective. The exponent $a(\varphi)$ only depends on the Cartan matrix of the block to which φ belongs. Moreover $p^{a(\varphi)}$ is bounded by the vertex of the module affording φ, and equality holds in case that G is p-solvable. We give some evidence for the conjecture that $a(\varphi) = 0$ occurs if and only if φ belongs to a block of defect 0. Finally, we study indecomposable quasi-projective Brauer characters of a block B. This set is finite and corresponds to a minimal Hilbert basis of the rational cone defined by the Cartan matrix of B.

Keywords: block, defect, Cartan matrix, Brauer character, quasi-projective character, projective module

MSC2010: 20C20

1. Introduction

Throughout this paper let p always denote a prime and let G be a finite group. By $\mathrm{Irr}(G)$ resp. $\mathrm{Irr}(B)$ we denote the set of ordinary irreducible characters of G resp. of a p-block B, and by $\mathrm{IBr}_p(G)$ resp. $\mathrm{IBr}_p(B)$ that of irreducible p-Brauer characters with respect to a p-modular splitting system. We put $l(B) = |\,\mathrm{IBr}_p(B)|$. Finally, we write Φ_φ for the ordinary character associated to the projective cover of the module affording $\varphi \in \mathrm{IBr}_p(G)$. If χ is a generalized ordinary character of G, then χ° denotes

The first author was supported by China Scholarship Council (201608360074), the National Natural Science Foundation of China (11661042) and (11471054), and the Project (GJJ150347) from Educational Department of Jiangxi Province.