On the moduli space of polygons with area center

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Abstract

A point p is said to be an area center of a polygon if all of the triangles composed of p and its edges have one and the same area. We construct a moduli space AC_n of such n-gons and study its geometry and arithmetic. For every $n \geq 5$, the moduli space is proved to be a rational complete intersection subvariety in $\mathbb{A}^n$. With the help of some subvarieties of low degree in AC_n, we also find a unified method of construction of good-looking polygons with area center.
keywords: polygon; area center; Chebyshev variety; rational point

0 Introduction

Let G be a barycenter of a triangle. Then the areas of three triangles made up of G and its edges are one and the same, as is shown by an elementary argument. In view of this fact, G is entitled to be called the *area center* of the triangle. A general n-gon, however, does not always have an area center (see Proposition 1.3). Hence it will be natural to consider when an n-gon has an area center. This seemingly innocent problem is, unexpectedly, found to have a intimate connection with the theory of Chebyshev varieties V_n developed in [2], [3]. In the present paper, we investigate the geometry and arithmetic of the moduli space, called AC_n, of n-gons with the origin as an area center. Among other things we show that AC_n is a rational complete intersection variety of codimension three in $\mathbb{A}^n$. This fact is proved by constructing a Groebner basis for its defining ideal, which in turn provides us with a parametrization of simple form for any n. Every parameter, however, does not always correspond to an n-gon with an area center of good shape. Here again we find that the family of linear subvarieties of a Chebyshev variety, constructed in [3], plays a crucial role to specify *good-looking* n-gons. Several examples of such n-gons are illustrated in the final section. Actually we need a slight generalization of the construction, and we introduce the set of strings of parentheses as well as those with *bra-ket*