Journal of Algebra 497 (2018) 337–362

Contents lists available at ScienceDirect

Journal of Algebra

www.elsevier.com/locate/jalgebra

The interplay between Steinberg algebras and skew rings ☆

Viviane Maria Beuter [a,b,*], Daniel Gonçalves [c]

[a] *Pós-Graduação em Matemática Pura e Aplicada, Universidade Federal de Santa Catarina, Florianópolis, 88040-900, Brazil*
[b] *Departamento de Matemática, Universidade do Estado de Santa Catarina, Joinville, 89219-710, Brazil*
[c] *Departamento de Matemática, Universidade Federal de Santa Catarina, Florianópolis, 88040-900, Brazil*

ARTICLE INFO

Article history:
Received 4 June 2017
Available online 1 December 2017
Communicated by Nicolás Andruskiewitsch

MSC:
16W22
16W55
22A22
47L40

Keywords:
Steinberg algebra
Partial skew group rings
Partial skew inverse semigroup rings
Diagonal preserving isomorphisms
Continuous orbit equivalence of partial actions

ABSTRACT

We study the interplay between Steinberg algebras and skew rings: For a partial action of a group in a Hausdorff, locally compact, totally disconnected topological space, we realize the associated partial skew group ring as a Steinberg algebra (over the transformation groupoid attached to the partial action). We then apply this realization to characterize diagonal preserving isomorphisms of partial skew group rings, over commutative algebras, in terms of continuous orbit equivalence of the associated partial actions. Finally, we show that any Steinberg algebra, associated to a Hausdorff ample groupoid, can be seen as a partial skew inverse semigroup ring.

☆ The second author was partially supported by CNPq – Conselho Nacional de Desenvolvimento Científico e Tecnológico – Brazil, grant number 304513/2014-8.
* Corresponding author.
E-mail addresses: vivibeuter@gmail.com (V.M. Beuter), daemig@gmail.com (D. Gonçalves).

https://doi.org/10.1016/j.jalgebra.2017.11.013
0021-8693/

1. Introduction

The notion of crossed product by a partial action has its origin in the operator algebra context, more precisely with the work of Exel (see [12]), on crossed products by a partial automorphism. Over the years the theory of partial actions has proved to be an important tool in the study of C*-algebras, and in particular of C*-algebras associated to dynamical systems and combinatorial objects, see for example [13–16,24,25] for a small glimpse of the theory developed.

In 2005, Dokuchaev and Exel (see [10]) started the study of partial skew group rings (an "algebraisation" of the operator theory concept of partial crossed products). Following the steps of the operator theory counter-part, partial skew rings are becoming an important tool in the study of algebras, in particular algebras associated with combinatorial objects. For example, Leavitt path algebras were realized as partial skew group rings in [22], and simplicity and chain conditions for partial skew group rings were studied in [21,23] and [28], respectively. Very recently, see [11], Dokuchaev and Exel studied the ideal structure of the partial skew group ring $\mathcal{L}_c(X) \rtimes_\alpha G$, where G is a discrete group, X is a locally compact, totally disconnected space X, and $\mathcal{L}_c(X)$ is the algebra consisting of all locally constant, compactly supported functions on X taking values in a given field K.

Steinberg algebras were independently introduced by Steinberg in [32] and by Clark et al. in [6]. They are the "algebraisation" of Renault's C*-algebras of groupoids. Even more than partial skew group rings, the development of the theory of Steinberg algebras has attracted a lot of attention lately. In particular, Steinberg algebras include the Kumjian–Pask algebras of higher-rank graphs introduced in [2] (which in turn include Leavitt path algebras). See [7], [8] and [33] for a few examples of the development of the theory.

It is our goal in this paper to link the theory of partial skew rings with the theory of Steinberg algebras, in the same way that the theory of partial crossed products is linked to Renault's theory of groupoid C*-algebras. In particular we provide an "algebraisation" of the result of Abadie, see [1], that shows that any partial crossed product, associated to a partial action on a topological space, can be seen as a groupoid C*-algebra. The algebraic version of this theorem permits us to join results of Li (see [27]), about continuous orbit equivalence of partial actions on topological spaces, and results of Carlsen and Rout (see [5]), about diagonal-preserving isomorphism between Steinberg algebras, to present results regarding diagonal preserving isomorphisms of partial skew group ring over commutative algebras.

To complete the interplay between Steinberg algebras and skew rings, we show an "algebraisation" of [29, Theorem 3.3.1] and [30, Theorem 8.1]: Any Steinberg algebra (associated to a Hausdorff ample groupoid) can be seen as a partial skew inverse semigroup ring (partial skew inverse semigroup rings generalize partial skew group rings and were defined by Exel and Vieira in [20], based on work by Nándor Sieben, see [31]). It is interesting to point out that the definition of a partial skew inverse semigroup ring