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Upper bounds for the dominant dimension of Nakayama and related algebras

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ABSTRACT

Optimal upper bounds are provided for the dominant dimensions of Nakayama algebras and more general algebras A with an idempotent e such that there is a minimal faithful injective–projective module eA and such that eAe is a Nakayama algebra. This answers a question of Abrar and proves a conjecture of Yamagata for monomial algebras.

Introduction

The dominant dimension domdim(A) of a finite dimensional algebra A is defined as follows: Let

$$0 \to A \to I_0 \to I_1 \to I_2 \to \ldots$$

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be a minimal injective resolution of the right regular module A. If I_0 is not projective, we set domdim(A):=0, and otherwise

$$\text{domdim}(A) := \sup\{n | I_i \text{ is projective for } i = 0, 1, ..., n\} + 1.$$

Note that the dominant dimension is invariant under Morita equivalence and also under field extensions (see [19] Lemma 5). Thus we can assume from now on that all algebras are basic and split over the field unless stated otherwise. For this reason we assume throughout this article that algebras are given by quiver and relations if not stated otherwise. One of the most famous conjectures in the representation theory of finite dimensional algebras is the Nakayama conjecture. This conjecture states that the dominant dimension of a non-selfinjective finite dimensional algebra is always finite (see [20]). A stronger conjecture was given in [24], where Yamagata conjectured that the dominant dimension is bounded by a function depending on the number of simple modules of a non-selfinjective algebra. Since the finiteness of the dominant dimension of a non-selfinjective algebra follows as a corollary of the finiteness of the finitistic dimension, the Nakayama conjecture is true for many classes of algebras. Examples include algebras with representation dimension smaller than or equal to 3 (see [14]). In contrast to that, explicit optimal bounds or values for the dominant dimension are rarely known for given classes of algebras. Here and in the following, an optimal bound denotes a bound on the dominant dimension such that the value of this bound is also attained in the given class of algebras. This leads to the following problem:

Problem. For a given class of connected non-selfinjective algebras, find optimal bounds for the dominant dimensions.

In [1] Theorem 1.2.3, Abrar shows that the dominant dimension of connected quiver algebras with an acyclic quiver is bound by the number of projective–injective indecomposable modules and that this bound is optimal for this class of algebras. Recall that Nakayama algebras are defined as algebras having the property that every indecomposable projective left or right module is uniserial, see for example [22] chapter I.10. for more information on Nakayama algebras. One conjecture about the optimal bound of the dominant dimension for non-selfinjective Nakayama algebras was given by Abrar in [1] as Conjecture 4.3.21:

Conjecture (Abrar). *Let A be a non-selfinjective Nakayama algebra with $n \geq 3$ simple modules. Then*

$$domdim(A) \leq 2n - 3.$$

In [1], Abrar calculated the dominant dimension for many Nakayama algebras and there the biggest value attained by a non-selfinjective Nakayama algebra with n simple