Journal of Algebra 496 (2018) 292–314

Contents lists available at ScienceDirect

Journal of Algebra

www.elsevier.com/locate/jalgebra

Affine cellularity of affine Yokonuma–Hecke algebras

Weideng Cui

School of Mathematics, Shandong University, Jinan, Shandong 250100, PR China

ARTICLE INFO

Article history:
Received 23 March 2017
Available online 3 November 2017
Communicated by Changchang Xi

MSC:
20C08
16E10

Keywords:
Affine Yokonuma–Hecke algebras
Affine cellular algebras
Matrix algebras
Global dimension

ABSTRACT

We establish an explicit algebra isomorphism between the affine Yokonuma–Hecke algebra $\widehat{Y}_{r,n}(q)$ and a direct sum of matrix algebras with coefficients in tensor products of affine Hecke algebras of type A. As an application of this result, we show that $\widehat{Y}_{r,n}(q)$ is affine cellular in the sense of Koenig and Xi, and further prove that it has finite global dimension when the parameter q is not a root of the Poincaré polynomial. As another application, we also recover the modular representation theory of $\widehat{Y}_{r,n}(q)$ previously obtained in [7].

1. Introduction

1.1. Inspired by Lusztig's work on the structure of cells and the based ring of affine Hecke algebras, Koenig and Xi [20] recently defined the notion of affine cellularity to generalize the notion of cellular algebras [9] to algebras of not necessarily finite dimension over a Noetherian domain k. Extended affine Hecke algebras of type A and affine Temperley–Lieb algebras were proved to be affine cellular in [20]. Further examples of affine cellular algebras include affine Hecke algebras of rank two with generic parame-

E-mail address: cwdeng@amss.ac.cn.

https://doi.org/10.1016/j.jalgebra.2017.10.014
0021-8693/

ters [10], KLR algebras of finite type [17], BLN algebras ([5] and [27]) and affine q-Schur algebras [6].

1.2. Yokonuma–Hecke algebras of general types were first introduced in the sixties by Yokonuma [32]. In the late 1990s and early 2000s, a new presentation of the Yokonuma–Hecke algebra $Y_{r,n}(q)$ of type A was given by Juyumaya and Kannan [12,14], which has been widely used for studying this algebra since then.

In recent years, many people are interested in the algebra $Y_{r,n}(q)$ from different perspectives. Some people studied $Y_{r,n}(q)$ in order to construct its associated knot invariant; see [13], [15] and [1] and so on. Others are particularly interested in the representation theory of $Y_{r,n}(q)$. Chlouveraki and Poulain d'Andecy [2] gave explicit formulas for all irreducible representations of $Y_{r,n}(q)$ over $\mathbb{C}(q)$ and obtained a semisimplicity criterion for it. Moreover, they defined a new kind of algebras, called affine Yokonuma–Hecke algebras and denoted by $\widehat{Y}_{r,n}(q)$. In their subsequent paper [3], they studied the representation theory of the affine Yokonuma–Hecke algebra $\widehat{Y}_{r,n}(q)$ and the cyclotomic Yokonuma–Hecke algebra $Y_{r,n}^{d}(q)$. In particular, they gave the classification of irreducible representations of $Y_{r,n}^{d}(q)$ in the generic semisimple case.

Besides, Jacon and Poulain d'Andecy [11] (see also [8]) gave an explicit algebraic isomorphism between the Yokonuma–Hecke algebra $Y_{r,n}(q)$ and a direct sum of matrix algebras over tensor products of Iwahori–Hecke algebras of type A, which is in fact a special case of the results by G. Lusztig [26, Section 34]. This allows them to give a description of the modular representation theory of $Y_{r,n}(q)$ and a complete classification of all Markov traces for it. Recently, Chlouveraki and Sécherre [4] proved that the affine Yokonuma–Hecke algebra is a particular case of the pro-p-Iwahori–Hecke algebras.

In [7], we have established an equivalence between a module category of $\widehat{Y}_{r,n}(q)$ (resp. $Y_{r,n}^{d}(q)$) and its suitable counterpart for a direct sum of tensor products of affine Hecke algebras of type A (resp. cyclotomic Hecke algebras), which allows us to give the classification of simple modules of $\widehat{Y}_{r,n}(q)$ and $Y_{r,n}^{d}(q)$ over an algebraically closed field of characteristic p such that p does not divide r.

1.3. Since the affine Hecke algebra of type A is affine cellular, it is natural to try to show that the affine Yokonuma–Hecke algebra $\widehat{Y}_{r,n}(q)$ is also affine cellular. In this paper, we will prove this fact by constructing an explicit algebra isomorphism between the affine Yokonuma–Hecke algebra $\widehat{Y}_{r,n}(q)$ and a direct sum of matrix algebras with coefficients in tensor products of various affine Hecke algebras of type A. As another application, we also recover the modular representation theory of $\widehat{Y}_{r,n}(q)$ previously obtained in [7].

This paper is organized as follows. In Section 2, we recall Koenig and Xi's results on affine cellular algebras, and then review the axiomatic approach to studying them presented in [6]. In Section 3, we give another presentation of the affine Yokonuma–Hecke algebra $\widehat{Y}_{r,n}(q)$. In Section 4, inspired by the work of Lusztig in [26], we give the construction of an algebra $\widehat{E}_{r,n}$, which is in fact a direct sum of matrix algebras with coefficients in tensor products of extended affine Hecke algebras of type A. Moreover,