Journal of Algebra 495 (2018) 81–113

Contents lists available at ScienceDirect

Journal of Algebra

www.elsevier.com/locate/jalgebra

The ideals of the slice Burnside p-biset functor ☆

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ARTICLE INFO

Article history:
Received 2 May 2017
Available online 31 October 2017
Communicated by Michel Broué

MSC:
18A25
19A22
20J15

Keywords:
Green biset functor
Slice Burnside ring
B-group

ABSTRACT

Let G be a finite group and $\mathbb{K}$ be a field of characteristic zero. Our purpose is to investigate the ideals of the slice Burnside functor $\mathbb{K}\Xi$. It turns out that they are the subfunctors F of $\mathbb{K}\Xi$ such that for any finite group G, the evaluation $F(G)$ is an ideal of the algebra $\mathbb{K}\Xi(G)$. This allows for a determination of the full lattice of ideals of the slice Burnside p-biset functor $\mathbb{K}\Xi_p$.

1. Introduction

The biset category $\mathcal{C}$ of finite groups has all finite groups as objects, the group of morphisms from a finite group G to a finite group H is the double Burnside group $B(H, G)$, i.e. the Grothendieck group of (H, G)-bisets. In particular the endomorphism ring of a finite group G is the double Burnside ring $B(G, G)$.

☆ This work is part of my doctoral thesis under Oumar Diankha (UCAD-Dakar-Senegal) and Serge Bouc (UPJV-Amiens-France).

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https://doi.org/10.1016/j.jalgebra.2017.10.010
0021-8693/© 2017 Published by Elsevier Inc.

A biset functor is an additive functor over this preadditive category, with values in abelian groups, and biset functors form an abelian category $\mathcal{F}$. More generally, one can extend the morphisms to $RB(H, G) = R \otimes_{\mathbb{Z}} B(H, G)$, where R is a commutative ring, and consider the R-linear functors with values in the category of R-modules. Thus, one obtain an R-linear abelian category $\mathcal{F}_R$.

A fundamental example of biset functor is the Burnside functor: it can be viewed as the representable functor $RB(1, -)$, or the Yoneda functor corresponding to the trivial group.

In particular it is a projective object of the category $\mathcal{F}_R$, which allows by the Yoneda–Dress construction to build enough projective objects on the category $\mathcal{F}_R$.

Moreover, the Burnside functor has a multiplicative structure which endows it with a structure of Green biset functor, and the modules over this Green functor are the biset functors.

We have a good parametrization of the isomorphism classes of simple RB-modules by isomorphism classes of pairs (H, V), where H is a finite group and V is a simple $R\mathrm{Out}(H)$-module. To each such (H, V) corresponds the isomorphism class of $S_{H,V}$, where $S_{H,V}(G)$ is the quotient of $L_{H,V}(G) = RB(G, H) \otimes_{RB(H,H)} V$ by

$$J_{H,V}(G) = \{\sum_i \phi_i \otimes v_i \in L_{H,V}(G) \mid \forall \psi \in \mathbb{K}B(H, G),\ \sum_i (\psi\phi_i).v_i = 0\}.$$

An important property of H is that it is a minimal group for $S_{H,V}$, and all the minimal groups for a simple biset functor are isomorphic.

Note that in general, the explicit computation of the evaluation $S_{H,V}(G)$ of a simple functor or more generally of a simple module over a Green biset functor is not easy (cf. [9] and [5]). The study of the functor $\mathbb{K}B$ (cf. [1]) where $\mathbb{K}$ is a field of characteristic zero, has allowed for an explicit description of some simple biset functors by the introduction of a new class of finite groups, the **B**-groups.

In this paper, we consider the slice Burnside ring introduced in ([3]). It is an analogue of the classical Burnside ring constructed from the morphisms of G-sets instead the G-sets themselves, and it shares most of its properties. In particular, as already shown by Serge Bouc (see [3] for more complete description), the slice Burnside ring is a commutative ring, which is free of finite rank as a $\mathbb{Z}$-module, and it becomes a split semisimple $\mathbb{Q}$-algebra, after tensoring with $\mathbb{Q}$. The correspondence which assigns to each finite group its slice Burnside ring has a natural biset functor structure, for which it becomes a Green biset functor.

We restrict our attention to the subcategory $\mathcal{C}_p$ of $\mathcal{C}$ whose objects are all the finite p-groups (for a given prime number p). To achieve the description of the ideals of the slice Burnside p-biset functor over a field of characteristic zero, we introduce the concept of **T**-slices instead of **B**-groups. In studying the ideals of the slice Burnside p-biset functor, we find another counterexample to Serge Bouc's conjecture saying that the minimal groups for a simple module over a Green biset functor form a single isomorphism class.