Some characterizations of finite σ-soluble $P\sigma T$-groups

Alexander N. Skiba

PII: S0021-8693(17)30573-2
DOI: https://doi.org/10.1016/j.jalgebra.2017.11.009
Reference: YJABR 16434

To appear in: *Journal of Algebra*

Received date: 2 April 2017

Please cite this article in press as: A.N. Skiba, Some characterizations of finite σ-soluble $P\sigma T$-groups, *J. Algebra* (2018), https://doi.org/10.1016/j.jalgebra.2017.11.009

Some characterizations of finite σ-soluble $P\sigma T$-groups

-In memory of Jim Beidleman

Alexander N. Skiba
Department of Mathematics and Technologies of Programming, Francisk Skorina Gomel State University, Gomel 246019, Belarus
E-mail: alexander.skiba49@gmail.com

Abstract

Let $\sigma = \{\sigma_i | i \in I\}$ be some partition of the set of all primes $\mathbb{P}$ and G a finite group. G is said to be *σ-soluble* if every chief factor H/K of G is a σ_i-group for some $i = i(H/K)$.

A set $\mathcal{H}$ of subgroups of G is said to be a *complete Hall σ-set* of G if every member $\neq 1$ of $\mathcal{H}$ is a Hall σ_i-subgroup of G for some $\sigma_i \in \sigma$ and $\mathcal{H}$ contains exactly one Hall σ_i-subgroup of G for every $i \in I$ such that $\sigma_i \cap \pi(G) \neq \emptyset$. A subgroup A of G is said to be *σ-permutable* in G if G has a complete Hall σ-set $\mathcal{H}$ such that $AH^x = H^xA$ for all $x \in G$ and all $H \in \mathcal{H}$.

We obtain characterizations of finite σ-soluble groups G in which σ-permutability is a transitive relation in G.

1 Introduction

Throughout this paper, all groups are finite and G always denotes a finite group. Moreover, $\mathbb{P}$ is the set of all primes, $\pi = \{p_1, \ldots, p_n\} \subseteq \mathbb{P}$ and $\pi' = \mathbb{P} \setminus \pi$. If n is an integer, the symbol $\pi(n)$ denotes the set of all primes dividing n; as usual, $\pi(G) = \pi(|G|)$, the set of all primes dividing the order of G. G is said to be a D_π-group if G possesses a Hall π-subgroup E and every π-subgroup of G is contained in some conjugate of E.

In what follows, σ is some partition of $\mathbb{P}$, that is, $\sigma = \{\sigma_i | i \in I\}$, where $\mathbb{P} = \cup_{i \in I} \sigma_i$ and $\sigma_i \cap \sigma_j = \emptyset$ for all $i \neq j$; Π is always supposed to be a subset of the set σ and $\Pi' = \sigma \setminus \Pi$.

By the analogy with the notation $\pi(n)$, we write $\sigma(n)$ to denote the set $\{\sigma_i | \sigma_i \cap \pi(n) \neq \emptyset\}$; $\sigma(G) = \sigma(|G|)$. G is said to be: *σ-primary* [20] if G is a σ_i-group for some i; *σ-decomposable* [21] or *σ-nilpotent* [14] if $G = G_1 \times \cdots \times G_n$ for some σ-primary groups $G_1, \ldots, G_n$; *σ-soluble* [20] if every chief factor of G is σ-primary; a *σ-full group of Sylow type* [20] if every subgroup E of G is a D_{σ_i}-group for every $\sigma_i \in \sigma(E)$. Note in passing, that every σ-soluble group is a σ-full group of Sylow type [22].

[0]Keywords: finite group, σ-permutable subgroup, $P\sigma T$-group, σ-soluble group, σ-nilpotent group.
[0]Mathematics Subject Classification (2010): 20D10, 20D15, 20D30