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BOUND FOR THE NUMBER OF ONE-DIMENSIONAL FIBERS OF A PROJECTIVE MORPHISM

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Abstract. Given a birational parameterization $\phi : \mathbb{P}^2_k \dashrightarrow \mathbb{P}^3_k$ of an algebraic surface $\mathscr{S} \subset \mathbb{P}^3_k$, we bound the number of one-dimensional fibers of the canonical projection of the graph of ϕ onto its image.

Keyword: *Fibers of morphism, implicitization, parameterization surfaces, elimination theory, approximation complex, geometric modeling.*

1. Introduction

Rational algebraic surfaces can be described in several ways, the most common being parametric and implicit representations. Parametric representations describe such surface as the image of a rational map, whereas implicit representations describe it as the zero locus of a certain algebraic equation, e.g. as the zeros of a polynomial. Both representations have a wide range of applications in Computer Aided Geometric Design, and depending on the problem one needs to solve, one or the other might be better suited. It is thus interesting to be able to pass from parametric representations to implicit equations. This is a classical problem and there are numerous approaches to its solution. For a good historical overview on this subject we refer the reader to [SC95] and [Cox01].

The implicitization problem has been widely studied and is always an active research area. It is basically an elimination problem, and thus can be solved by methods based on Gröbner basis computations. However, it is known to be quite slow in practice and hence is rarely used in geometric modeling (see, e.g., [Hof89]). A more common method for finding the implicit equation is to eliminate by computing the resultant of the polynomials (see, e.g., [Jou96, Section 5.3.17], [Bus01]). But in many applications, the resultant vanishes identically due to the presence of base points. In consequence, the search of formulas for implicitization rational surfaces with base points is a very active area of research due to the fact that, in practical industrial design, base points show up quite frequently. In [MC92], a perturbation is applied to resultants in order to obtain a nonzero multiple of the implicit equation. Many other approaches have been done in this direction. More recently, by using the approximation complexes that were defined by Herzog, Simis and Vasconcelos in [HSV82], a new method for computing implicitizations as the determinant of the approximation complexes have been described in [BJ03, BCD03, BC05, Cha06, BCJ09].

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