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# General offender theory

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ABSTRACT

We present an offender theory that is symmetric in offender and offended group and also a replacement theorem that does not need that the groups in question are abelian. We then use this theory to define variations of Thompson and Baumann subgroups and prove a general Baumann argument.
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The use of best offenders[1] is one of the most important tools in the $p$-local analysis of finite groups. One of the reasons is that in a finite group $G$ the Thompson subgroup of a Sylow $p$-subgroup of $G$ is generated by subgroups that act as best offenders on each (elementary) abelian normal $p$-subgroup of $G$, see 3.2. Another reason is that pairs $(G, V)$, where $G$ is a finite group with $O_p(G) = 1$ and $V$ is a finite faithful $\mathbb{F}_pG$-module, can be completely classified provided that $G$ is generated by best offenders on $V$, see for example [2].

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[1] See below for the definition of best offender.

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In this paper we want to develop a general theory of offenders and then show that some basic concepts and results about best offenders and the Thompson subgroup can be generalized. We have chosen the replacement theorem, the definition of the Thompson and Baumann subgroup, and the Baumann argument as examples to demonstrate this.

In more than one way our approach is more general than the one used in the literature. There one needs a finite group $G$ and a finite abelian $p$-group $V$ together with an action of $G$ on $V$ if one wants to speak of offenders (on $V$). That is, the situation is not symmetric in $G$ and $V$.

There is one remarkable exception. Already in [1] the authors have dropped the assumption that offenders or the group $V$ have to be abelian, but kept the asymmetric set-up that $G$ has to act on $V$. They also have been more interested in what we will call global $\alpha$-offenders. Their considerations inspired us to proceed with our more general approach. See also our comment in the next section.

In contrast to the above, our approach is symmetric in $G$ and $V$. That is, we drop that $G$ has to act on $V$ together with the assumption that $V$ or the offenders on $V$ have to be abelian. This then also allows the case $G = V$, which we find particularly interesting.

## 1. The general set-up, definitions and results

We start with a group $H$ and a fixed pair $(\alpha, \beta)$ of positive real numbers. For any finite subgroup $V \leq H$ we define a measure $\| \, \|_V$ on (the finite subgroups of) $H$ depending on $V$ and $(\alpha, \beta)$:

$$\|A\|_V := |A|^\alpha |C_V(A)|^\beta \text{ for all finite subgroups } A \leq H.$$

As one can see, the group $H$ is only present to allow to speak of "the centralizer of $A$ in $V$".

The measure $\| \, \|_V$ is called $\alpha$-measure if $\beta = 1$, and classical if $\alpha = \beta = 1$.

Let $G$ be a second finite subgroup of $H$ and $\| \, \|_V$ be the measure defined above with respect to $V$ and $(\alpha, \beta)$. Then the $G$-dual of $\| \, \|_V$, denoted by $\| \, \|_G^*$, is the measure on $H$ defined with respect to $G$ and $(\beta, \alpha)$. That is,

$$\|A\|_G^* := |A|^\beta |C_G(A)|^\alpha \text{ for all finite subgroups } A \leq H.$$

**General set-up:** $G$ and $V$ are two finite subgroups of a given group $H$, and $\| \, \|_V$ is a fixed measure on $H$ (defined with respect to $V$ and a pair $(\alpha, \beta)$ of positive real numbers) together with its $G$-dual $\| \, \|_G^*$.

Observe that this set-up is symmetric in $G$ and $V$ since $\| \, \|_V$ is the $V$-dual of $\| \, \|_G^*$. General offender theory is now the investigation of $G$ and $V$ by means of $\| \, \|_V$ and its $G$-dual $\| \, \|_G^*$.

This set-up generalizes all situations where offenders are used in literature. For example, if $V$ is a normal subgroup of a finite group $G$ then one can use $H := G$; and if

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