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QUANTUM QUASI-SHUFFLE ALGEBRAS II

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To Christian Kassel with admiration

ABSTRACT. Using the concept of mixable shuffles, we formulate explicitly the quantum quasi-shuffle product, as well as the subalgebra generated by primitive elements of the quantum quasi-shuffle bialgebra. We construct a braided coalgebra which is dual to the quantum quasi-shuffle algebra. We provide representations of quantum quasi-shuffle algebras on commutative braided Rota-Baxter algebras. As an application, we establish formal power series whose terms come from a special representation of the quasi-shuffle algebra on polynomial algebra and whose evaluations at 1 are the multiple q-zeta values.

1. INTRODUCTION

In [28], Ree introduced the shuffle algebra which has been studied extensively during the past fifty years. The shuffle product is carried out on the tensor space $T(V)$ of a vector space V by using the shuffle rule. Its natural generalization is the quasi-shuffle product where V is moreover an associative algebra and the new product on $T(V)$ involves both of the shuffle product and the multiplication of V. Quasi-shuffle algebras first arose in the work of Newman and Radford [26] for the study of cofree irreducible Hopf algebras built on associative algebras, where they were constructed by the universal property of cofree pointed irreducible coalgebras. Later, they were rediscovered independently by other mathematicians with various motivations. In 2000, motivated by his work on multiple zeta values, Hoffman defined the quasi-shuffle algebra by an inductive formula ([14]). In the same year, Guo and Keigher introduced the mixable shuffle algebra by using an explicit formula in their study of Rota-Baxter algebras ([10] and [12]). After these seminal works, the research of quasi-shuffle algebras become active. Besides their own interest, quasi-shuffle algebras have many significant applications in other branches of mathematics, such as multiple zeta values ([15]), Rota-Baxter algebras ([10] and [5]), and commutative tridendriform algebras ([23]). They also appear in the study of shuffle identities between Feynman graphs ([22]).

For both of physical and mathematical considerations, people want to deform or quantize some important algebra structures. The most famous class of examples is given by quantum groups which are introduced by Drinfeld [4] and Jimbo [19]. To people's surprise, there is an implicit but significant connection between quantum groups and shuffle algebras. Rosso [29] constructed the quantization of

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