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Classification of asexual diploid organisms by means of strongly isotopic evolution algebras defined over any field

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Evolution algebras were introduced into Genetics to deal with the mechanism of inheritance of asexual organisms. Their distribution into isotopism classes is uniquely related with the mutation of alleles in non-Mendelian Genetics. This paper deals with such a distribution by means of Computational Algebraic Geometry. We focus in particular on the two-dimensional case, which is related to the asexual reproduction processes of diploid organisms. Specifically, we determine the existence of four isotopism classes, whatever the base field is, and we characterize the corresponding isomorphism classes.

1. Introduction

In the middle of the twentieth century, nonassociative algebras were introduced in Genetics by Etherington [14–16] in order to endow Mendel's laws with a mathematical

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formulation that simulates the sexual reproduction and the mechanism of inheritance of an organism by considering the fusion of gametes into a zygote as an algebraic multiplication whose structure constants determine the probability distribution of the gametic output. Much more recently, in order to deal with asexual reproduction processes, Tian and Vojtechovsky [25,26] introduced evolution algebras as a type of genetic algebra that makes possible to deal algebraically with the self-reproduction of alleles in non-Mendelian Genetics. The fundamentals of such algebras have been being developed in the last years with no probabilistic restrictions on the structure constants [3,5,7,8,12,13,21–24]. Nowadays, evolution algebras also constitute a fundamental connection between algebra, dynamic systems, Markov processes, Knot Theory, Graph Theory and Group Theory [20,25].

A main problem in the theory of evolution algebras is their distribution into isomorphism and isotopism classes. On the one hand, the mentioned distribution into isomorphism classes has already been dealt with for two-dimensional evolution algebras over the complex field [4,9] and for nilpotent evolution algebras of dimension up to four over arbitrary fields [18]. On the other hand, isotopisms have emerged as an interesting tool to simulate mutations in genetic algebras. In this regard, Holgate and Campos [6,19] had already considered isotopisms of genetic algebras as a way to formulate mathematically the mutation of alleles in the inheritance process. They showed indeed that certain known families of genetic algebras are isotopic. Nevertheless, to the best of the authors knowledge, isotopisms have not yet been considered in case of dealing with evolution algebras. The main goal of this paper is to delve further into this aspect.

The paper is organized as follows. In Section 2, we indicate some preliminary concepts and results on isotopisms of algebras, genetic algebras and Computational Algebraic Geometry which will be used in the rest of the paper. Section 3 deals with the distribution of finite-dimensional evolution algebras over any base field into isotopism classes according to their structure tuples and to the dimension of their annihilators. Particularly, we determine the existence of four isotopism classes of two-dimensional evolution algebras, whatever the base field is. After that, we focus in Section 4 on the corresponding distribution of two-dimensional evolution algebras over any base field into isomorphism classes. In Section 5, we illustrate with the three-dimensional case how to deal with the distribution into isotopism classes of evolution algebras of higher dimensions. Finally, since this paper has a high dependence on notation, a glossary of symbols is shown in Appendix A.

2. Preliminaries

In this section we expose some basic concepts and results on isotopisms of algebras, genetic algebras and Computational Algebraic Geometry that are used throughout the paper. For more details about these topics we refer, respectively, to the manuscripts of Albert [1], Wörz-Busekros [27], Tian [25] and Cox et al. [10].