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**GLOBAL IN TIME STRICHARTZ ESTIMATES FOR THE FRACTIONAL
SCHRÖDINGER EQUATIONS ON ASYMPTOTICALLY EUCLIDEAN
MANIFOLDS**

VAN DUONG DINH

ABSTRACT. In this paper, we prove global in time Strichartz estimates for the fractional Schrödinger operators, namely $e^{-it\Lambda_g^\sigma}$ with $\sigma \in (0, \infty) \setminus \{1\}$ and $\Lambda_g := \sqrt{-\Delta_g}$ where Δ_g is the Laplace-Beltrami operator on asymptotically Euclidean manifolds (\mathbb{R}^d, g) . Let $f_0 \in C_0^\infty(\mathbb{R})$ be a smooth cutoff equal 1 near zero. We firstly show that the high frequency part $(1 - f_0)(P)e^{-it\Lambda_g^\sigma}$ satisfies global in time Strichartz estimates as on \mathbb{R}^d of dimension $d \geq 2$ inside a compact set under non-trapping condition. On the other hand, under the moderate trapping assumption (1.12), the high frequency part also satisfies the global in time Strichartz estimates outside a compact set. We next prove that the low frequency part $f_0(P)e^{-it\Lambda_g^\sigma}$ satisfies global in time Strichartz estimates as on \mathbb{R}^d of dimension $d \geq 3$ without using any geometric assumption on g . As a byproduct, we prove global in time Strichartz estimates for the fractional Schrödinger and wave equations on (\mathbb{R}^d, g) , $d \geq 3$ under non-trapping condition.

1. INTRODUCTION

Let (M, g) be a d -dimensional Riemannian manifold. We consider the time dependent fractional Schrödinger equation on (M, g) , namely

$$i\partial_t u - \Lambda_g^\sigma u = 0, \quad u|_{t=0} = u_0, \quad (1.1)$$

with $\sigma \in (0, \infty) \setminus \{1\}$, $\Lambda_g = \sqrt{-\Delta_g}$ where Δ_g is the Laplace-Beltrami operator associated to the metric g . The fractional Schrödinger equation (1.1) arises in many physical contexts. When $\sigma \in (0, 2) \setminus \{1\}$, the fractional Schrödinger equation was discovered by N. Laskin (see [27], [28]) as a result of extending the Feynman path integral, from the Brownian-like to Lévy-like quantum mechanical paths. This type of equation also appears in the water wave models (see [22], [32]). When $\sigma = 2$, it corresponds to the well-known Schrödinger equation. In the case $\sigma = 4$, it is the fourth-order Schrödinger equation introduced by Karpman [24] and Karpman and Shagalov [25] to take into account the role of small fourth-order dispersion terms in the propagation of intense laser beams in a bulk medium with Kerr nonlinearity.

When $M = \mathbb{R}^d$ and $g = \text{Id}$, i.e. the flat Euclidean space, the solution to (1.1) enjoys the following global in time Strichartz estimates (see [16]),

$$\|u\|_{L^p(\mathbb{R}, L^q(\mathbb{R}^d))} \lesssim \|u_0\|_{\dot{H}^{\gamma_{p,q}}(\mathbb{R}^d)},$$

where (p, q) satisfies the fractional admissible condition, i.e.

$$p \in [2, \infty], \quad q \in [2, \infty), \quad (p, q, d) \neq (2, \infty, 2), \quad \frac{2}{p} + \frac{d}{q} \leq \frac{d}{2}, \quad (1.2)$$

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Key words and phrases. Global in time Strichartz estimate; fractional Schrödinger equation; Littlewood-Paley decomposition, Isozaki-Kitada parametrix.

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