Accepted Manuscript

Global in time Strichartz estimates for the fractional Schrödinger equations on asymptotically Euclidean manifolds

Van Duong Dinh

PII: S0022-1236(18)30260-X
DOI: https://doi.org/10.1016/j.jfa.2018.07.006
Reference: YJFAN 8059

To appear in: *Journal of Functional Analysis*

Received date: 13 May 2017
Accepted date: 13 July 2018

GLOBAL IN TIME STRICHARTZ ESTIMATES FOR THE FRACTIONAL SCHRÖDINGER EQUATIONS ON ASYMPTOTICALLY EUCLIDEAN MANIFOLDS

VAN DUONG DINH

Abstract. In this paper, we prove global in time Strichartz estimates for the fractional Schrödinger operators, namely $e^{-it\Lambda_g^\sigma}$ with $\sigma \in (0,\infty)\backslash\{1\}$ and $\Lambda_g := \sqrt{-\Delta_g}$ where Δ_g is the Laplace-Beltrami operator on asymptotically Euclidean manifolds $(\mathbb{R}^d, g)$. Let $f_0 \in C_0^\infty(\mathbb{R})$ be a smooth cutoff equal 1 near zero. We firstly show that the high frequency part $(1-f_0)(P)e^{-it\Lambda_g^\sigma}$ satisfies global in time Strichartz estimates as on $\mathbb{R}^d$ of dimension $d \geq 2$ inside a compact set under non-trapping condition. On the other hand, under the moderate trapping assumption (1.12), the high frequency part also satisfies the global in time Strichartz estimates outside a compact set. We next prove that the low frequency part $f_0(P)e^{-it\Lambda_g^\sigma}$ satisfies global in time Strichartz estimates as on $\mathbb{R}^d$ of dimension $d \geq 3$ without using any geometric assumption on g. As a byproduct, we prove global in time Strichartz estimates for the fractional Schrödinger and wave equations on $(\mathbb{R}^d, g), d \geq 3$ under non-trapping condition.

1. Introduction

Let (M, g) be a d-dimensional Riemannian manifold. We consider the time dependent fractional Schrödinger equation on (M, g), namely

$$i\partial_t u - \Lambda_g^\sigma u = 0, \quad u_{|t=0} = u_0, \tag{1.1}$$

with $\sigma \in (0,\infty)\backslash\{1\}, \Lambda_g = \sqrt{-\Delta_g}$ where Δ_g is the Laplace-Beltrami operator associated to the metric g. The fractional Schrödinger equation (1.1) arises in many physical contexts. When $\sigma \in (0,2)\backslash\{1\}$, the fractional Schrödinger equation was discovered by N. Laskin (see [27], [28]) as a result of extending the Feynman path integral, from the Brownian-like to Lévy-like quantum mechanical paths. This type of equation also appears in the water wave models (see [22], [32]). When $\sigma = 2$, it corresponds to the well-known Schrödinger equation. In the case $\sigma = 4$, it is the fourth-order Schrödinger equation introduced by Karpman [24] and Karpman and Shagalov [25] to take into account the role of small fourth-order dispersion terms in the propagation of intense laser beams in a bulk medium with Kerr nonlinearity.

When $M = \mathbb{R}^d$ and $g = \mathrm{Id}$, i.e. the flat Euclidean space, the solution to (1.1) enjoys the following global in time Strichartz estimates (see [16]),

$$\|u\|_{L^p(\mathbb{R}, L^q(\mathbb{R}^d))} \lesssim \|u_0\|_{\dot{H}^{\gamma_{p,q}}(\mathbb{R}^d)},$$

where (p, q) satisfies the fractional admissible condition, i.e.

$$p \in [2,\infty], \quad q \in [2,\infty), \quad (p,q,d) \neq (2,\infty,2), \quad \frac{2}{p} + \frac{d}{q} \leq \frac{d}{2}, \tag{1.2}$$

2010 *Mathematics Subject Classification.* 35G20, 35G25.

Key words and phrases. Global in time Strichartz estimate; fractional Schrödinger equation; Littlewood-Paley decomposition, Isozaki-Kitada parametrix.