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Path-dependent Hamilton–Jacobi equations in infinite dimensions

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ABSTRACT

We propose notions of minimax and viscosity solutions for a class of fully nonlinear path-dependent PDEs with nonlinear, monotone, and coercive operators on Hilbert space. Our main result is well-posedness (existence, uniqueness, and stability) for minimax solutions. A particular novelty is a suitable combination of minimax and viscosity solution techniques in the proof of the comparison principle. One of the main difficulties, the lack of compactness in infinite-dimensional Hilbert spaces, is circumvented by working with suitable compact subsets of our path space. As an application, our theory makes it possible to employ the dynamic programming approach to study optimal control problems for a fairly general class of (delay) evolution equations in the variational framework. Furthermore, differential games associated to such evolution equations can be investigated following the Krasovski –Subbotin approach similarly as in finite dimensions.

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Contents

1.	Introduction	2097
1.1.	Related research	2099
1.2.	Our approach and the main difficulties	2100
1.3.	Notions, main results, and methodology in a simplified setting	2102
1.4.	Organization of the rest of the paper	2105
2.	Setting and preliminary results	2105
2.1.	Path space and related topologies	2106
2.2.	The operator A , related trajectory spaces, and evolution equations	2107
2.3.	Chain rule and standard derivatives	2112
2.4.	Functional chain rule and path derivatives	2114
3.	Path-dependent Hamilton–Jacobi equations	2115
3.1.	Minimax solutions: global version of the notion	2116
3.2.	Minimax solutions: infinitesimal version of the notion	2118
4.	Comparison	2120
5.	Existence and uniqueness	2123
6.	Stability	2130
7.	Applications to optimal control	2132
8.	Applications to differential games	2138
Appendix A.	Properties of solution sets of evolution equations	2147
Appendix B.	Other notions of solutions	2154
B.1.	Classical solutions	2154
B.2.	Viscosity solutions	2156
References	2157

1. Introduction

Let $V \subseteq H \subseteq V^*$ be a Gelfand triple, i.e., V is a separable reflexive Banach space with a continuous and dense embedding into a Hilbert space H . Moreover, this embedding is assumed to be compact. We study fully nonlinear so-called path-dependent PDEs (PPDEs) of the form

$$\partial_t u(t, x) - \langle A(t, x(t)), \partial_x u(t, x) \rangle + F(t, x, \partial_x u(t, x)) = 0, \tag{1.1}$$

$$(t, x) \in [0, T] \times C([0, T], H).$$

Here, $A(t, \cdot) : V \rightarrow V^*$, $t \in [0, T]$, are nonlinear, monotone, coercive operators. The derivatives $\partial_t u$ and $\partial_x u$ are certain path derivatives on the path space $C([0, T], H)$. A definition will be provided later. Note that they are not Fréchet derivatives. Let us also mention that any kind of solution u of (1.1) should at least be *non-anticipating*, i.e., for every $x, y \in C([0, T], H)$, whenever $x = y$ on $[0, t]$, we have $u(t, x) = u(t, y)$.

A particular example of (1.1) is the Bellman equation associated to distributed control problems for divergence-form quasi-linear parabolic PDEs of order $2m$ on some bounded domain $G \subset \mathbb{R}^n$ with smooth boundary ∂G , e.g., the problem of minimizing a cost functional

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