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Path-dependent Hamilton–Jacobi equations in infinite dimensions

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We propose notions of minimax and viscosity solutions for a class of fully nonlinear path-dependent PDEs with nonlinear, monotone, and coercive operators on Hilbert space. Our main result is well-posedness (existence, uniqueness, and stability) for minimax solutions. A particular novelty is a suitable combination of minimax and viscosity solution techniques in the proof of the comparison principle. One of the main difficulties, the lack of compactness in infinite-dimensional Hilbert spaces, is circumvented by working with suitable compact subsets of our path space. As an application, our theory makes it possible to employ the dynamic programming approach to study optimal control problems for a fairly general class of (delay) evolution equations in the variational framework. Furthermore, differential games associated to such evolution equations can be investigated following the Krasovskiĭ–Subbotin approach similarly as in finite dimensions.

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Contents

1. Introduction

Let $V \subseteq H \subseteq V^*$ be a Gelfand triple, i.e., V is a separable reflexive Banach space with a continuous and dense embedding into a Hilbert space H. Moreover, this embedding is assumed to be compact. We study fully nonlinear so-called path-dependent PDEs (PPDEs) of the form

$$\begin{aligned} &\partial_t u(t,x) - \langle A(t,x(t)), \partial_x u(t,x)\rangle + F(t,x,\partial_x u(t,x)) = 0, \\ &\qquad\qquad (t,x) \in [0,T) \times C([0,T],H). \end{aligned} \tag{1.1}$$

Here, $A(t,\cdot) : V \to V^*$, $t \in [0,T]$, are nonlinear, monotone, coercive operators. The derivatives $\partial_t u$ and $\partial_x u$ are certain path derivatives on the path space $C([0,T],H)$. A definition will be provided later. Note that they are not Fréchet derivatives. Let us also mention that any kind of solution u of (1.1) should at least be *non-anticipating*, i.e., for every x, $y \in C([0,T],H)$, whenever $x = y$ on $[0,t]$, we have $u(t,x) = u(t,y)$.

A particular example of (1.1) is the Bellman equation associated to distributed control problems for divergence-form quasi-linear parabolic PDEs of order $2m$ on some bounded domain $G \subset \mathbb{R}^n$ with smooth boundary ∂G, e.g., the problem of minimizing a cost functional