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Existence and nonexistence of positive solutions to some fully nonlinear equation in one dimension

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A B S T R A C T

In this paper, we consider the existence (and nonexistence) of solutions to

$$-\mathcal{M}^{\pm}_{\lambda,\Lambda}(u'') + V(x)u = f(u) \quad \text{in } \mathbf{R}$$

where $\mathcal{M}^{+}_{\lambda,\Lambda}$ and $\mathcal{M}^{-}_{\lambda,\Lambda}$ denote the Pucci operators with $0 < \lambda \le \Lambda < \infty$, $V(x)$ is a bounded function, $f(s)$ is a continuous function and its typical example is a power-type nonlinearity $f(s) = |s|^{p-1}s$ ($p > 1$). In particular, we are interested in positive solutions which decay at infinity, and the existence (and nonexistence) of such solutions is proved.

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1. Introduction

In this paper, we study the existence and nonexistence of solutions to the following nonlinear differential equations

$$-\mathcal{M}^{\pm}_{\lambda,\Lambda}(u'') + V(x)u = f(u) \quad \text{in } \mathbf{R}, \quad u > 0 \quad \text{in } \mathbf{R}, \quad \lim_{|x|\to\infty} u(x) = 0. \tag{1.1}$$

Here V and f are given functions, $0 < \lambda \leq \Lambda < \infty$ constants and $\mathcal{M}^{\pm}_{\lambda,\Lambda}(s)$ the Pucci operators defined by

$$\mathcal{M}^{+}_{\lambda,\Lambda}(s) := \begin{cases} \Lambda s & \text{if } s \geq 0, \\ \lambda s & \text{if } s < 0, \end{cases} \qquad \mathcal{M}^{-}_{\lambda,\Lambda}(s) := \begin{cases} \lambda s & \text{if } s \geq 0, \\ \Lambda s & \text{if } s < 0. \end{cases}$$

We remark that when $\lambda = \Lambda$, one has $\mathcal{M}^{\pm}_{\lambda,\Lambda}(u'') = \lambda u''$.

One of motivations to study equations like (1.1) is to see to what extent the properties and the results in the semilinear case can be generalized to the fully nonlinear case. When $\lambda = \Lambda$, (1.1) is well studied and it is proved that (1.1) has a solution for various $V(x)$ and $f(s)$ by critical point theory. Here we refer to [11,12] and references therein.

On the other hand, when $\lambda \neq \Lambda$, (1.1) is not studied well. In [7], instead of (1.1), the authors study the existence of positive radial solutions of

$$-\mathcal{M}^{\pm}_{\lambda,\Lambda}(D^2 u) + \gamma u = f(u) \quad \text{in } B_R(0) \subset \mathbf{R}^N, \quad u = 0 \quad \text{on } \partial B_R(0) \tag{1.2}$$

as well as

$$-\mathcal{M}^{\pm}_{\lambda,\Lambda}(D^2 u) + u = u^p \quad \text{in } \mathbf{R}^N.$$

Here $N \geq 3$, $0 \leq \gamma$ and $1 < p < p^{\pm}_{*}$ where $p^{\pm}_{*}$ are critical exponents for $\mathcal{M}^{\pm}_{\lambda,\Lambda}$ (see also [1,3,5,6]). Recently, in [9], the authors show the existence of infinitely many radial solutions of (1.2) when $\gamma = 0$ and $f(s) = |s|^{p-1}s$. Moreover, in [9], the inhomogeneous case is also considered and the existence of infinitely many solutions is shown on a bounded annulus.

In this paper, we aim to treat the inhomogeneous equation on the unbounded domain $\mathbf{R}$. We emphasis that in general the existence of solutions to (1.1) is delicate when the equation is inhomogeneous and the domain is unbounded. Indeed, we shall prove the nonexistence result when $V(x)$ is monotone. See Theorem 1.2 below.

We first deal with the existence result. For $V(x)$, we assume

(V1) $V \in W^{1,\infty}(\mathbf{R})$ and $0 < \inf_{\mathbf{R}} V =: V_0$.
(V2) For a.a. $x \in (-\infty, 0)$ and a.a. $y \in (0,\infty)$, $V'(x) \leq 0 \leq V'(y)$.
(V3) $V(0) \leq V_\infty := \lim_{|x|\to\infty} V(x)$ and there exist $C_0, \xi_0 > 0$ such that

$$(\text{for } \mathcal{M}^{+}_{\lambda,\Lambda}) \quad (0 \leq) V_\infty - V(x) \leq C_0 \exp\left(-2\sqrt{\frac{V_\infty}{\Lambda} + \xi_0}\,|x|\right) \quad \text{for all } x \in \mathbf{R},$$