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On quotients of spaces with Ricci curvature bounded below

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A B S T R A C T

Let (M, g) be a smooth Riemannian manifold and G a compact Lie group acting on M effectively and by isometries. It is well known that a lower bound of the sectional curvature of (M, g) is again a bound for the curvature of the quotient space, which is an Alexandrov space of curvature bounded below. Moreover, the analogous stability property holds for metric foliations and submersions.

The goal of the paper is to prove the corresponding stability properties for synthetic Ricci curvature lower bounds. Specifically, we show that such stability holds for quotients of $\mathsf{RCD}^*(K, N)$-spaces, under isomorphic compact group actions and more generally under metric-measure foliations and submetries. An $\mathsf{RCD}^*(K, N)$-space is a metric measure space with an upper dimension bound N and weighted Ricci curvature bounded below by K in a generalized sense. In particular, this shows that if (M, g) has Ricci curvature bounded below by $K \in \mathbb{R}$ and dimension N, then the quotient space is an

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$\mathsf{RCD}^*(K,N)$-space. Additionally, we tackle the same problem for the $\mathsf{CD}/\mathsf{CD}^*$ and MCP curvature-dimension conditions.
We provide as well geometric applications which include: A generalization of Kobayashi's Classification Theorem of homogeneous manifolds to $\mathsf{RCD}^*(K,N)$-spaces with *essential minimal dimension* $n \leq N$; a structure theorem for $\mathsf{RCD}^*(K,N)$-spaces admitting actions by *large (compact) groups*; and geometric rigidity results for orbifolds such as Cheng's Maximal Diameter and Maximal Volume Rigidity Theorems.
Finally, in two appendices we apply the methods of the paper to study quotients by isometric group actions of discrete spaces and of (super-)Ricci flows.

Contents

1. Introduction

Studying the geometry of isometry groups has proven to be advantageous for the understanding of Riemannian manifolds. For instance, this point of view has been par-