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# A HELSON MATRIX WITH EXPLICIT EIGENVALUE ASYMPTOTICS

NAZAR MIHEISI AND ALEXANDER PUSHNITSKI

ABSTRACT. A Helson matrix (also known as a multiplicative Hankel matrix) is an infinite matrix with entries  $\{a(jk)\}$  for  $j, k \geq 1$ . Here the  $(j, k)$ 'th term depends on the product  $jk$ . We study a self-adjoint Helson matrix for a particular sequence  $a(j) = (\sqrt{j} \log j (\log \log j)^\alpha)^{-1}$ ,  $j \geq 3$ , where  $\alpha > 0$ , and prove that it is compact and that its eigenvalues obey the asymptotics  $\lambda_n \sim \varkappa(\alpha)/n^\alpha$  as  $n \rightarrow \infty$ , with an explicit constant  $\varkappa(\alpha)$ . We also establish some intermediate results (of an independent interest) which give a connection between the spectral properties of a Helson matrix and those of its continuous analogue, which we call the integral Helson operator.

## 1. INTRODUCTION

**1.1. Background: Hankel matrices.** We start our discussion by recalling relevant facts from the theory of Hankel matrices. Let  $\{b(j)\}_{j=0}^\infty$  be a sequence of complex numbers. A *Hankel matrix* is an infinite matrix of the form

$$H(b) = \{b(j+k)\}_{j,k=0}^\infty,$$

considered as a linear operator in  $\ell^2(\mathbb{Z}_+)$ ,  $\mathbb{Z}_+ = \{0, 1, 2, \dots\}$ . One of the key examples of Hankel matrices is the *Hilbert matrix*, which corresponds to the choice  $b(j) = 1/(j+1)$ . It is well known that the Hilbert matrix is bounded (but not compact). From the boundedness of the Hilbert matrix by a simple argument one obtains

$$b(j) = o(1/j), \quad j \rightarrow \infty \quad \Rightarrow \quad H(b) \text{ is compact.}$$

A natural family of compact self-adjoint Hankel operators of this class was considered in [13]. To state this result, we need some notation. For a compact self-adjoint operator  $A$ , let us denote by  $\{\lambda_n^+(A)\}_{n=1}^\infty$  the non-increasing sequence of positive eigenvalues (enumerated with multiplicities taken into account), and let  $\lambda_n^-(A) = \lambda_n^+(-A)$ .

**Theorem A.** [13] *Let  $b(j)$  be a sequence of real numbers defined by*

$$b(j) = 1/(j(\log j)^\alpha), \quad j \geq 2;$$

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