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F.E. Castillo-Sántos, P.N. Dowling, H. Fetter, M. Japón, C.J. Lennard et al.

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**NEAR-INFINITY CONCENTRATED NORMS AND THE FIXED  
POINT PROPERTY FOR NONEXPANSIVE MAPS ON CLOSED,  
BOUNDED, CONVEX SETS**

F.E. CASTILLO-SÁNTOS, P.N. DOWLING, H. FETTER, M. JAPÓN, C.J. LENNARD,  
B. SIMS, B. TURETT

**ABSTRACT.** In this paper we define the concept of a near-infinity concentrated norm on a Banach space  $X$  with a boundedly complete Schauder basis. When  $\|\cdot\|$  is such a norm, we prove that  $(X, \|\cdot\|)$  has the fixed point property (FPP); that is, every nonexpansive self-mapping defined on a closed, bounded, convex subset has a fixed point. In particular, P.K. Lin's norm in  $\ell_1$  [14] and the norm  $\nu_p(\cdot)$  (with  $p = (p_n)$  and  $\lim_n p_n = 1$ ) introduced in [3] are examples of near-infinity concentrated norms. When  $\nu_p(\cdot)$  is equivalent to the  $\ell_1$ -norm, it was an open problem as to whether  $(\ell_1, \nu_p(\cdot))$  had the FPP. We prove that the norm  $\nu_p(\cdot)$  always generates a nonreflexive Banach space  $X = \mathbb{R} \oplus_{p_1} (\mathbb{R} \oplus_{p_2} (\mathbb{R} \oplus_{p_3} \dots))$  satisfying the FPP, regardless of whether  $\nu_p(\cdot)$  is equivalent to the  $\ell_1$ -norm. We also obtain some stability results.

1. INTRODUCTION AND PRELIMINARIES

Let  $(X, \|\cdot\|)$  be a Banach space and  $C$  a subset of  $X$ . A mapping  $T : C \rightarrow C$  is said to be nonexpansive if  $\|Tx - Ty\| \leq \|x - y\|$  for every  $x, y \in C$ . The Banach space  $X$  endowed with the norm  $\|\cdot\|$  has the fixed point property (FPP) if every nonexpansive mapping defined from a closed bounded convex subset  $C$  of  $X$  into itself has a fixed point. This property is not preserved by isomorphism, that is, it strongly depends on the underlying norm [14]. There is a wide literature relating geometric properties of reflexive Banach spaces with the fulfilment of the fixed point property (see, for instance, the monographs [9], [13] and the references therein).

The Banach space  $\ell_1$  endowed with its standard norm  $\|\cdot\|_1$  is a classical example of a nonreflexive Banach space that fails to have the FPP. It is possible to “perturb” this  $(\ell_1, \|\cdot\|_1)$ -example to obtain other Banach spaces that fail to have the FPP. One such class of Banach spaces are those that contain asymptotically isometric copies of  $\ell_1$ .

Recall that a Banach space  $(X, \|\cdot\|)$  contains an asymptotically isometric copy (a.i.c.) of  $\ell_1$  if there are a sequence  $(x_n) \subset X$  and a decreasing sequence  $(\epsilon_n) \subset (0, 1)$  with  $\lim_n \epsilon_n = 0$  such that

$$\sum_{n=1}^{\infty} (1 - \epsilon_n) |t_n| \leq \left\| \sum_{n=1}^{\infty} t_n x_n \right\| \leq \sum_{n=1}^{\infty} |t_n|$$

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