Journal of Functional Analysis 275 (2018) 698–711

Contents lists available at ScienceDirect

Journal of Functional Analysis

www.elsevier.com/locate/jfa

Estimates near the origin for functional calculus on analytic semigroups

I. Chalendar [a,*], J. Esterle [b], J.R. Partington [c]

[a] *Université Paris Est Marne-la-Vallée, 5 bd Descartes, Champs-sur-Marne, 77454 Marne-la-Vallée, cedex 2, France*
[b] *IMB, UMR 5251, Université de Bordeaux, 351 cours de la Libération, 33405 Talence Cedex, France*
[c] *School of Mathematics, University of Leeds, Leeds LS2 9JT, UK*

ARTICLE INFO

Article history:
Received 18 September 2017
Accepted 16 March 2018
Available online 19 March 2018
Communicated by K. Seip

MSC:
primary 47D03, 46J40, 46H30
secondary 30A42, 47A60

Keywords:
Strongly continuous semigroup
Functional calculus
Fourier–Borel transform
Analytic semigroup

ABSTRACT

This paper provides sharp lower estimates near the origin for the functional calculus $F(-uA)$ of a generator A of an operator semigroup defined on a sector; here F is given as the Fourier–Borel transform of an analytic functional. The results are linked to the existence of an identity element in the Banach algebra generated by the semigroup. Both the quasinilpotent and non-quasinilpotent cases are considered, and sharp results are proved extending many in the literature.

* Corresponding author.
E-mail addresses: isabelle.chalendar@u-pem.fr (I. Chalendar), jean.esterle@math.u-bordeaux1.fr (J. Esterle), J.R.Partington@leeds.ac.uk (J.R. Partington).

https://doi.org/10.1016/j.jfa.2018.03.012

1. Introduction

The purpose of this paper is to prove results concerning norm estimates in analytic semigroups which complement the results proved in [3] for semigroups defined on $\mathbb{R}^+$. For good references on analytic semigroups we recommend the books [4,11]: we note that analytic semigroups of operators are norm-continuous in the open sector of the plane on which they are defined, and thus also act as semigroups by multiplication on the Banach algebra that they generate. Thus we may easily pass from the language of operators to the language of Banach algebras.

In [1] the following result was proved for semigroups defined on the right-hand half-plane $\mathbb{C}_+$. Here, ρ denotes the spectral radius of an operator, and Rad denotes the radical of an algebra.

Theorem 1.1. *Let $(T(t))_{t\in\mathbb{C}_+}$ be an analytic non-quasinilpotent semigroup in a Banach algebra. Let $\mathcal{A}_T$ be the closed subalgebra generated by $(T(t))_{t\in\mathbb{C}_+}$ and let $\gamma > 0$. If there exists $t_0 > 0$ such that*

$$\sup_{t\in\mathbb{C}_+, |t|\le t_0} \rho(T(t) - T((\gamma+1)t)) < 2$$

then $\mathcal{A}_T/\operatorname{Rad}\mathcal{A}_T$ is unital, and the generator of $(\pi(T(t))_{t>0}$ is bounded, where $\pi : \mathcal{A}_T \to \mathcal{A}_T/\operatorname{Rad}\mathcal{A}_T$ denotes the canonical surjection.

This can be seen as a lower estimate for a functional calculus in $\mathcal{A}_T$, determined by $F(-A) = T(t) - T((\gamma+1)t)$, where $F : s \to e^{-s} - e^{-(\gamma+1)s}$ is the Laplace transform of the atomic measure $\delta_1 - \delta_{\gamma+1}$.

This approach was taken in [3] for semigroups defined on $\mathbb{R}_+$, and very general results were proved for both the quasinilpotent and non-quasinilpotent cases, providing extensions of results in [5,6,9] and elsewhere. For a detailed history of the subject, we refer to [2].

Fewer results are available for analytic semigroups, and virtually nothing involving a general functional calculus, although some dichotomy results are given in [2]. To prove more general results for analytic semigroups requires the notion of the Fourier–Borel transform of a distribution acting on analytic functions, and in Section 2 we define these transforms and the associated functional calculus.

In Section 3 we derive results in the non-quasinilpotent case, using the properties of the characters defined on the algebra $\mathcal{A}_T$, putting Theorem 1.1 in a much more general context. Note that the results we prove are sharp, as is shown in Example 3.6.

Finally, the more difficult case of quasinilpotent semigroups is treated in Section 4, adapting the complex variable methods introduced in [3]. Here there are additional technical difficulties involved in defining the functional calculus, since we now work with measures supported on compact subsets of $\mathbb{C}$.