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Spectral transition line for the extended Harper's model in the positive Lyapunov exponent regime

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A R T I C L E   I N F O

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We study the spectral transition line of the extended Harper's model in the positive Lyapunov exponent regime. We show that both pure point spectrum and purely singular continuous spectrum occur for dense subsets of frequencies on the transition line.

1. Introduction

Quasiperiodic Jacobi matrices arise naturally from the study of tight-binding electrons on a two-dimensional lattice exposed to a perpendicular magnetic field. The most prominent example of such operators is the Harper's equation, mathematically known as the almost Mathieu operator (AMO), acting on $l^2(\mathbb{Z})$, defined by (under a non-standard scaling),

$$(Hu)_n = \lambda(u_{n+1} + u_{n-1}) + 2\cos 2\pi(\theta + n\alpha)u_n. \tag{1.1}$$

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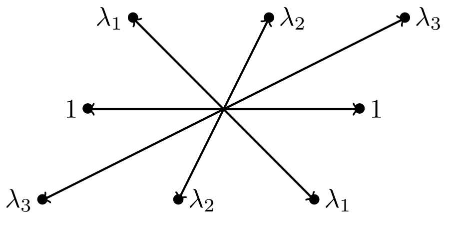
This paper considers a more general model that was introduced by D.J. Thouless in 1983 [19], which includes the AMO as a special case. The Hamiltonian of the extended Harper's model (EHM), denoted by $H_{\lambda,\alpha,\theta}$, is defined as follow.

$$(H_{\lambda,\alpha,\theta}u)_n = c_\lambda(\theta + n\alpha)u_{n+1} + \tilde{c}_\lambda(\theta + (n-1)\alpha)u_{n-1} + 2\cos 2\pi(\theta + n\alpha)u_n, \qquad (1.2)$$

acting on $l^2(\mathbb{Z})$, in which

$$\begin{cases} c_\lambda(\theta) = \lambda_1 e^{-2\pi i(\theta+\frac{\alpha}{2})} + \lambda_2 + \lambda_3 e^{2\pi i(\theta+\frac{\alpha}{2})} \\ \tilde{c}_\lambda(\theta) = \lambda_3 e^{-2\pi i(\theta+\frac{\alpha}{2})} + \lambda_2 + \lambda_1 e^{2\pi i(\theta+\frac{\alpha}{2})} \end{cases}$$

We refer to $\alpha \in \mathbb{T} = [0,1]$ as the frequency and let $\beta(\alpha)$ (see (2.6)) to be the upper exponent of exponential growth of continued fraction approximants. We also refer to $\theta \in \mathbb{T}$ as the phase and let $\lambda = (\lambda_1, \lambda_2, \lambda_3)$ be the coupling constant triple. The coupling constants are proportional to the probabilities an electron will hop to a corresponding neighboring site. Without loss of generality, we assume $0 \le \lambda_2$, $0 \le \lambda_1 + \lambda_3$ and at least one of λ_1, λ_2, λ_3 to be positive. While the AMO ($\lambda_1 = \lambda_3 = 0$) only takes nearest-neighbor hopping into account, the EHM includes next-nearest-neighbor hopping:



Nearest-neighbour hopping: encoded in λ_2 and 1,
Next nearest-neighbour hopping: encoded in λ_1 and λ_3.

In the past few years, there have been several remarkable developments on obtaining arithmetic spectral transition for concrete quasiperiodic Schrödinger operators. For the Maryland model, the spectral phase diagram was determined exactly for all α, θ in [14]. For the AMO, the spectral transition conjecture in α [12]: pure point spectrum (a.e. θ) for $\beta(\alpha) < -\ln\lambda$ (under our non-standard scaling) and purely singular continuous spectrum for $\beta(\alpha) > -\ln\lambda$, was recently proved in [4]. Later, universal (reflective) hierarchical structure of eigenfunctions was established in the localization regime [15,16] with also an arithmetic condition on θ. Even more recently, the spectral transition line $\beta(\alpha) = -\ln\lambda$ was studied in [3], where the authors showed that both pure point (for a.e. θ) and purely singular continuous spectrum are dense phenomena.

The extended Harper's model, as a prime example of quasiperiodic (non-Schrödinger) Jacobi matrix, has also attracted great attention from both mathematics and physics (see e.g. [5,6,19]) literature. Under the classical duality map $\lambda = (\lambda_1, \lambda_2, \lambda_3) \to$