Conic Kähler-Einstein metrics along simple normal crossing divisors on Fano manifolds

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CONIC KÄHLER-EINSTEIN METRICS ALONG SIMPLE NORMAL CROSSING DIVISORS ON FANO MANIFOLDS

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ABSTRACT. We prove that on one Kähler-Einstein Fano manifold without holomorphic vector fields, there exists a unique conical Kähler-Einstein metric along a simple normal crossing divisor with admissible prescribed cone angles. We also establish a curvature estimate for conic metrics along a simple normal crossing divisor which generalizes Li-Rubinstein's curvature estimate for one divisor case.

1. INTRODUCTION

Conic Kähler metrics are very useful in the study of Kähler geometry. Recently there are a lot of works on this topic [1] [4] [5] [15] [16] [17] [18] [22] [25] [26] [32] [38] [40] [41]. It played a crucial role in the solution to Yau-Tian-Donaldson Conjecture [37] [7] [8] [9] on Fano manifolds. In this solution one important part is the analysis of the continuity path of the cone angle, which has been studied in [15]. The idea is to investigate the behavior of the solutions to conical Kähler-Einstein metrics as the angle tends to 1. Then the stability condition guarantees that the solution could be extended, which generates a smooth Kähler-Einstein metric. On the other hand, we can consider the existence of conical Kähler-Einstein metrics assuming the existence of smooth Kähler-Einstein metrics, which could be studied along the continuity path of decreasing cone angles. Related works in this direction could be found in [22] [32] [40] in case of one smooth divisor. In [32], Song-Wang also considered the case of simple normal crossing divisors on toric Fano manifolds. In this paper, we consider the general situation of simple normal crossing divisors and our main result is as below, which could be thought as the generalization of Theorem 1.1 of [22]:

Theorem 1.1. *Given a Fano manifold (M, ω_0) without holomorphic vector fields where $[\omega_0] = c_1(M)$, which admits a Kähler-Einstein metric ω_{KE} satisfying $Ric(\omega_{KE}) = \omega_{KE}$. For one simple normal crossing divisor $D = \sum_{r=1}^{m} D_r$ where each D_r is semi-ample, and a sequence of positive rational numbers $c_1, \cdots, c_m$ which satisfy that*

$$\sum_{r=1}^{m} c_r[D_r] = c_1(M). \tag{1.1}$$

If for all $r = 1, \cdots, m$ it holds that $c_r > 1$ or

$$\lambda_r := \inf\{\lambda > 0 | \lambda K_M^{-1} - [D_r] > 0\} \geq \frac{n}{n+1}$$