ARTICLE IN PRESS YJFAN:7980

Journal of Functional Analysis ••• (••••) •••–•••

Contents lists available at ScienceDirect

Journal of Functional Analysis

www.elsevier.com/locate/jfa

Global dynamics of a classical Lotka–Volterra competition–diffusion–advection system ☆

Peng Zhou [a], Dongmei Xiao [b,*]

[a] Department of Mathematics, Shanghai Normal University, Shanghai 200234, PR China

[b] School of Mathematical Sciences, Shanghai Jiao Tong University, Shanghai 200240, PR China

A R T I C L E I N F O

Article history:
Received 28 August 2017
Accepted 1 March 2018
Available online xxxx
Communicated by Martin Hairer

MSC:
35K57
35K61
37C65
92D25

Keywords:
Competition–diffusion–advection
Monotone dynamical systems
Principal eigenvalue
Global stability

A B S T R A C T

In this paper, we study a classical two species Lotka–Volterra competition–diffusion–advection system, where the diffusion and advection rates of two competitors are supposed to be proportional. By employing the principal spectral theory, we first establish a key a priori estimate on the co-existence (positive) steady state, which is a powerful tool to link the local and global dynamics. We then further present a complete classification on all possible long-time dynamical behaviors by appealing to the theory of monotone dynamical systems. Lastly, we apply these results to a special situation where two species are competing for the same resources and obtain a sharp criteria in term of certain variable parameters for all kinds of global dynamics. This work gives a positive answer to the conjecture proposed by Lou et al. in [34] by considering a more general model under certain conditions, and also, can be seen as a further development of He and Ni [19] for competition–diffusion system, where we bring new ingredients

☆ Zhou's research is supported in part by the Program for Professor of Special Appointment (Eastern Scholar) at Shanghai Institutions of Higher Learning and the NSF of China (#11571363); and Xiao's research is supported by the NSF of China (#11431008).

* Corresponding author.
E-mail addresses: pengzhou011@126.com (P. Zhou), xiaodm@sjtu.edu.cn (D. Xiao).

https://doi.org/10.1016/j.jfa.2018.03.006

Please cite this article in press as: P. Zhou, D. Xiao, Global dynamics of a classical Lotka–Volterra competition–diffusion–advection system, J. Funct. Anal. (2018), https://doi.org/10.1016/j.jfa.2018.03.006

in the arguments to overcome the difficulty caused by the involvement of advection.

1. Introduction

It is a dream to obtain global dynamics from local dynamics in competitive Lotka–Volterra systems, see, e.g., [19,43,44] and references therein. In this paper, we are looking for conditions to realize this dream for the following classical Lotka–Volterra type competition–diffusion–advection system

$$\begin{cases} u_t = d_1\triangle u - \alpha_1\nabla\cdot[u\nabla P(x)] + u[r_1(x) - u - cv], & x\in\Omega,\ t>0,\\ v_t = d_2\triangle v - \alpha_2\nabla\cdot[v\nabla P(x)] + v[r_2(x) - bu - v], & x\in\Omega,\ t>0,\\ d_1\frac{\partial u}{\partial n} - \alpha_1 u\frac{\partial P(x)}{\partial n} = 0, & x\in\partial\Omega,\ t>0,\\ d_2\frac{\partial v}{\partial n} - \alpha_2 v\frac{\partial P(x)}{\partial n} = 0, & x\in\partial\Omega,\ t>0,\\ u(x,0) = u_0(x) \geq, \not\equiv 0, & x\in\Omega,\\ v(x,0) = v_0(x) \geq, \not\equiv 0, & x\in\Omega, \end{cases} \tag{1.1}$$

where $u(x,t)$ and $v(x,t)$ represent the population density of two competing species at location $x\in\Omega$ and time $t>0$, respectively; Ω, the habitat, is a bounded smooth domain in $\mathbb{R}^N$, $1\leqslant N\in\mathbb{Z}$; $d_1, d_2>0$ are random diffusion rates of species u and v, respectively; the non-constant function $P(x)\in C^2(\overline{\Omega})$ is used to specify the advective direction, and the advection rates of two species are denoted by $\alpha_1, \alpha_2>0$, respectively; $b, c>0$ measure the inter-specific competition ability, while the intra-specific competition coefficients are normalized by 1 here; $\triangle = \sum_{i=1}^N \frac{\partial^2}{\partial x_i^2}$ is the usual Laplace operator; n denotes the outward unit normal vector on the boundary $\partial\Omega$; $r_1(x)$ and $r_2(x)$ stand for the local carrying capacity or intrinsic growth rates of species u and v, respectively, which sometimes are also called environment/resource functions since they can reflect the environmental situations. The no-flux boundary conditions imposed above mean that no individuals can move in or out through the boundary of the habitat. For simplicity, we will assume, throughout this paper, that the initial functions $u_0(x)$ and $v_0(x)$ are nonnegative and nontrivial, i.e., not identically zero (otherwise, problem (1.1) will be reduced to a logistic type single species problem whose population dynamics is standard).

System (1.1) has important applications in several different biological scenarios. For example, by letting $r_1 = r_2 = r(x)$, $b = c = 1$ and $P(x) = r(x)$, one obtains the following competition model

Please cite this article in press as: P. Zhou, D. Xiao, Global dynamics of a classical Lotka–Volterra competition–diffusion–advection system, J. Funct. Anal. (2018), https://doi.org/10.1016/j.jfa.2018.03.006