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On fixed points of self maps of the free ball

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A R T I C L E I N F O

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A B S T R A C T

In this paper, we study the structure of the fixed point sets of noncommutative self maps of the free ball. We show that for such a map that fixes the origin the fixed point set on every level is the intersection of the ball with a linear subspace. We provide an application for the completely isometric isomorphism problem of multiplier algebras of noncommutative complete Pick spaces.

1. Introduction

Function theory and hyperbolic geometry of $\mathbb{B}_d$, the unit ball of $\mathbb{C}^d$, were studied extensively throughout the years, see for example [59] and [30]. The fact that $\mathbb{B}_d$ is the unit ball of a finite dimensional Hilbert space leads to a generalization of many classical results from the unit disc setting, like the Schwarz lemma and the Julia–Caratheodory–Wolff theorem (see [59] and [37] for details).

In operator algebra theory the Drury–Arveson space $\mathcal{H}^2_d$, the model space for commuting row contractions, is a reproducing kernel Hilbert space of analytic function on $\mathbb{B}_d$ with reproducing kernel $k_d(z,w) = \frac{1}{1-\langle z,w\rangle}$ (see [11] and [24]). This space is a complete Pick space, i.e., the multipliers of the Drury–Arveson space admit an interpolation theo-

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rem for matrix valued functions generalizing the classical Nevanlinna–Pick interpolation theorem in the unit disc. Let us write $\mathcal{M}_d$ for the algebra of multipliers on $\mathcal{H}_d^2$, it is a maximal abelian WOT-closed operator subalgebra of $B(\mathcal{H}_d^2)$ generated by the operators M_{z_j} of multiplication by coordinate functions. In [3, Theorem 8.2] it was shown that the Drury–Arveson space for $d = \infty$ is the universal complete Pick space, namely, if $\mathcal{H}$ is a separable complete Pick reproducing kernel Hilbert space on a set X with kernel k, then there exists an embedding $b \colon X \to \mathbb{B}_\infty$ and a nowhere vanishing function δ on X, such that $k(x,y) = \overline{\delta(x)}\delta(y)k_\infty(b(x), b(y))$ and $\mathcal{H}$ is isometrically embedded in $\delta\mathcal{H}_d^2$.

Let $V \subset \mathbb{B}_d$ be an analytic subvariety of $\mathbb{B}_d$ cut out by functions in $\mathcal{M}_d$. We can associate to it a reproducing kernel Hilbert space $\mathcal{H}_V$ spanned by kernel functions $k_d(\cdot, w)$ for $w \in V$. This space turns out to be a complete Pick space and the multiplier algebra $\mathcal{M}_V$ of $\mathcal{H}_V$ is completely isometrically isomorphic to $\mathcal{M}_d/\mathcal{I}_V$, where $\mathcal{I}_V$ is the WOT-closed ideal of functions vanishing on V. It is thus natural to ask to what extent does the algebra $\mathcal{M}_V$ determine the variety V and vice versa. The isomorphism problem for subvarieties of $\mathbb{B}_d$ cut out by multipliers of the Drury–Arveson space was studied by Davidson, Ramsey and Shalit. In [22] and [23] they studied the algebra $\mathcal{M}_V$ and its norm closed analog and proved that if $V, W \subset \mathfrak{B}_d$ are subvarieties of $\mathfrak{B}_d$, such that their affine span is all of $\mathbb{C}^d$, then $\mathcal{M}_V$ is completely isometrically isomorphic to $\mathcal{M}_W$ if and only if there is an automorphism of $\mathbb{B}_d$ mapping V onto W (see also [60] for a survey and more results on the commutative isomorphism problem). One of the main tools in the proof of the theorem is a theorem that appears both in [59] and [37] and states that the fixed point set of a self map of $\mathbb{B}_d$ is the intersection of $\mathbb{B}_d$ with an affine subspace.

The noncommutative (nc for short), or free functions were introduced by Taylor in [62] and [63]. Taylor's goal was to facilitate noncommutative functional calculus and thus he endeavored to give topological algebras analogous to the classical Frechet algebras of analytic functions on open domains in $\mathbb{C}^d$. Voiculescu in [68], [69], [70] and [71] developed the ideas of Taylor in the context of free probability. Helton, Klep, McCullough and Schweighofer applied noncommutative analysis in order to obtain dimension free relaxation of the LMI containment problem (see [32] and [33]). Their results were extended and improved upon by Davidson, Dor-On, Shalit and Solel in [18], Passer, Shalit and Solel in [47] and Fritz, Netzer and Thom in [29]. For other applications to free real algebraic geometry see for example [35] and [36]. More applications to free probability were provided by Belinschi, Popa and Vinnikov in [15] and Popa and Vinnikov in [48]. A fundamental book [38] on the properties of nc function was written by Kaliuzhnyi-Verbovetskyi and Vinnikov. The theory of bounded functions on free domains was studied by Agler and McCarthy in [2], [4], [5], [8], [7] and [6]. They have obtained interpolation and realization results with applications to H^∞ functional calculus on free analogs of polynomial polyhedra. Similar interpolation and realization results were obtained by Ball, Marx and Vinnikov in [14] and [13], where they have also developed the theory of noncommutative reproducing kernels and defined the complete Pick property for such kernels. Muhly

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