ARTICLE IN PRESS YJFAN:7995

Journal of Functional Analysis ••• (••••) •••–•••

Contents lists available at ScienceDirect

Journal of Functional Analysis

www.elsevier.com/locate/jfa

Accumulation of complex eigenvalues of a class of analytic operator functions

Christian Engström, Axel Torshage *

Department of Mathematics and Mathematical Statistics, Umeå University, SE-901 87 Umeå, Sweden

ARTICLE INFO

Article history:
Received 5 September 2017
Accepted 24 March 2018
Available online xxxx
Communicated by K. Seip

MSC:
47A56
47J10
47A10
47A12

Keywords:
Operator pencil
Spectral divisor
Numerical range
Non-linear spectral problem

ABSTRACT

For analytic operator functions, we prove accumulation of branches of complex eigenvalues to the essential spectrum. Moreover, we show minimality and completeness of the corresponding system of eigenvectors and associated vectors. These results are used to prove sufficient conditions for eigenvalue accumulation to the poles and to infinity of rational operator functions. Finally, an application of electromagnetic field theory is given.

1. Introduction

In recent years quantitative information on the discrete spectrum of non-selfadjoint operators has gained considerable interest [1,23,13,16,14]. In particularly, Pavlov's influential papers on accumulation of complex eigenvalues to the essential spectrum of

* Corresponding author.
E-mail addresses: christian.engstrom@math.umu.se (C. Engström), axel.torshage@math.unibe.ch (A. Torshage).

https://doi.org/10.1016/j.jfa.2018.03.019

Schrödinger operators with non-selfadjoint Robin boundary conditions [25–27] have been extended to magnetic Schrödinger operators [29] and to Schrödinger operators with a complex potential [3]. Importantly, the potential will in some cases also depend on time [28] and differential operators with time dependent coefficients are common in e.g. electromagnetics [4] and viscoelasticity [30]. In these cases, theory for operator functions with a non-linear dependence of the spectral parameter is used to determine the spectral properties. Krein & Langer [18] proved for selfadjoint quadratic operator polynomials $\lambda - A - \lambda^2 B$ with real numerical range, sufficient conditions for eigenvalue accumulation in terms of the numerical range of A and the numerical range of B. More recently, selfadjoint operator functions with real eigenvalues have been studied extensively [20,6,21,22,10]. Still, there have been very few results on accumulation of complex eigenvalues of operator functions with the notable exception [2] that proved accumulation of eigenvalues for a quadratic operator polynomial from elasticity theory.

In this paper, we study polynomial and rational operator functions. These types of functions share spectral properties with a linear non-selfadjoint operator called the linearized operator. However, since in non-trivial cases the linearized operator is not a relatively compact perturbation of a selfadjoint operator, the known results for non-selfadjoint operators can not be used to prove accumulation of eigenvalues. Therefore, we will explore factorization results for holomorphic operator functions [24, Chapter III]. We extend theory based on those factorization results with the aim to provide sufficient conditions for eigenvalue accumulation of a class of unbounded rational operator functions. A major difficulty is that in theory based on the factorization of operator polynomials one must know specific properties of the numerical range to prove accumulation of eigenvalues. The main contribution of [2] is that they for the particular quadratic operator polynomial prove the existence of a bounded part of the numerical range that is separated from rest of the numerical range. In [11], we proposed a new type of enclosure of the numerical range that can be used to determine the number of components of the numerical range. Importantly, it is much easier to determine the number of components of this enclosure than to directly determine the number of components of the numerical range. Therefore, the new enclosure of the numerical range is used to prove accumulation of eigenvalues, and to prove completeness of the corresponding system of eigenvectors and associated vectors.

The paper is organized as follows. In Section 2, we present the basic notation and definitions used in the paper.

In Section 3, we consider for a class of bounded analytic operator functions minimality and completeness of the set of eigenvectors and associated vectors corresponding to a branch of eigenvalues. In particular, we generalize [24, Theorem 22.13] to cases with a spectral divisor of order larger than one. Our main results are Theorem 3.9 and Theorem 3.10.

In Section 4, we study rational operator functions whose values are Fredholm operators. The main results are Theorem 4.6 and Theorem 4.8, which utilize the results

Please cite this article in press as: C. Engström, A. Torshage, Accumulation of complex eigenvalues of a class of analytic operator functions, J. Funct. Anal. (2018), https://doi.org/10.1016/j.jfa.2018.03.019