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Fractional div-curl quantities and applications to nonlocal geometric equations

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We investigate a fractional notion of gradient and divergence operator. We generalize the div-curl estimate by Coifman–Lions–Meyer–Semmes to fractional div-curl quantities, obtaining, in particular, a nonlocal version of Wente's lemma.
We demonstrate how these quantities appear naturally in nonlocal geometric equations, which can be used to obtain a theory for fractional harmonic maps analogous to the local theory. Firstly, regarding fractional harmonic maps into spheres, we obtain a conservation law analogous to Shatah's conservation law and give a new regularity proof analogous to Hélein's for harmonic maps into spheres.
Secondly, we prove regularity for solutions to critical systems with nonlocal antisymmetric potentials on the right-hand side. Since the half-harmonic map equation into general target manifolds has this form, as a corollary, we obtain a new proof of the regularity of half-harmonic maps into general target manifolds following closely Rivière's celebrated argument in the local case.
Lastly, the fractional div-curl quantities provide also a new, simpler, proof for Hölder continuity of $W^{s,n/s}$-harmonic maps into spheres and we extend this to an argument for $W^{s,n/s}$-harmonic maps into homogeneous targets. This is an analogue of Strzelecki's and Toro–Wang's proof for

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n-harmonic maps into spheres and homogeneous target manifolds, respectively.

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Contents

1. Introduction

Products of divergence-free and curl-free vector fields, the so-called div-curl-quantities, play a fundamental role in Geometric Analysis. They appear, for example, in the theory of compensated compactness in the form of the div-curl Lemma: let $L^2(\bigwedge^1 \mathbb{R}^n)$ be the L^2-space of 1-forms on $\mathbb{R}^n$, or equivalently the space of vector fields $L^2(\mathbb{R}^n, \mathbb{R}^n)$. Given two sequences $\{F_k\}_{k\in\mathbb{N}}, \{G_k\}_{k\in\mathbb{N}}$ in $L^2(\bigwedge^1 \mathbb{R}^n)$ which weakly converge in $L^2(\bigwedge^1 \mathbb{R}^n)$ to F and G, respectively. In general, there is no reason that the product converges

$$F_k \cdot G_k \xrightarrow{k\to\infty} F \cdot G \quad \text{in } \mathcal{D}'(\mathbb{R}^n). \tag{1.1}$$

If we know, however, that (in distributional sense) $\operatorname{div}(F_k) = 0$ and $\operatorname{curl}(G_k) = 0$, or more generally assuming compactness of $\operatorname{div}(F_k)$ and $\operatorname{curl}(G_k)$ in H^{-1}, then (1.1) indeed holds true. This phenomenon is known as compensated compactness and its theory was developed by Murat and Tartar in the late seventies [28,29,47–49], see also the more recent [8,10].

In [9] Coifman, Lions, Meyer, and Semmes found a relation between div-curl quantities and the Hardy space $\mathcal{H}^1(\mathbb{R}^n)$ (for a definition see Section 6).

Theorem 1.1 *(Coifman–Lions–Meyer–Semmes). Let $F \in L^p(\bigwedge^1 \mathbb{R}^n)$ and $g \in \dot{W}^{1,p'}(\mathbb{R}^n)$ where $p \in (1,\infty)$ and $p' = \frac{p}{p-1}$. Then, if*

$$\operatorname{div} F = 0,$$

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