ARTICLE IN PRESS YJFAN:7906

Journal of Functional Analysis ••• (••••) •••–•••

Contents lists available at ScienceDirect

Journal of Functional Analysis

www.elsevier.com/locate/jfa

Bergman inner functions and m-hypercontractions

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ARTICLE INFO

Article history:
Received 17 July 2017
Accepted 27 October 2017
Available online xxxx
Communicated by K. Seip

MSC:
primary 47A13
secondary 47A20, 47A45, 47A48

Keywords:
Bergman inner functions
Hypercontractions
Characteristic functions
Wandering subspaces

ABSTRACT

We characterize operator-valued Bergman inner functions on the unit ball as functions admitting a suitable transfer function realization. Thus we extend corresponding one-variable results of Olofsson from the case of the unit disc to the unit ball. At the same time we associate with each m-hypercontraction $T \in L(H)^n$ a canonical Bergman inner function W_T and indicate a possible definition of a characteristic function θ_T for T.

1. Introduction

A commuting tuple $T = (T_1, \ldots, T_n) \in L(H)^n$ of bounded linear operators on a complex Hilbert space H is by definition a row contraction if the operator

$$H^n \to H, (x_i)_{1 \le i \le n} \mapsto \sum_{i=1}^{n} T_i x_i$$

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https://doi.org/10.1016/j.jfa.2017.10.018

is a contraction. A dilation result of Müller and Vasilescu [18], extended by Arveson [3], shows that a tuple $T \in L(H)^n$ is a row contraction if and only if T is up to unitary equivalence a compression of the direct sum $M_z \oplus U \in L(H(\mathbb{B}, \mathcal{D}) \oplus K)^n$ of a Drury–Arveson shift $M_z \in L(H(\mathbb{B}, \mathcal{D}))^n$ and a spherical unitary $U \in L(K)^n$ to one of its co-invariant subspaces. More precisely, let $\mathbb{B} \subset \mathbb{C}^n$ be the open Euclidean unit ball. Then by definition $M_z = (M_{z_1}, \ldots, M_{z_n}) \in L(H(\mathbb{B}, \mathcal{D}))^n$ is the tuple of multiplication operators with the coordinate functions on the $\mathcal{D}$-valued analytic functional Hilbert space $H(\mathbb{B}, \mathcal{D})$, with a suitable Hilbert space $\mathcal{D}$, given by the reproducing kernel

$$K : \mathbb{B} \times \mathbb{B} \to L(\mathcal{D}), (z, w) \mapsto \frac{1_{\mathcal{D}}}{1 - \langle z, w \rangle},$$

while a spherical unitary is a commuting tuple $U = (U_1, \ldots, U_n) \in L(K)^n$ of normal operators with $\sum_{i=1}^n U_i U_i^* = 1_K$. The same condition, but without the spherical unitary part $U \in L(K)^n$, characterizes precisely the row contractions $T \in L(H)^n$ which are pure in the sense that they satisfy a $C_{\cdot 0}$-condition of the form

$$\text{SOT} - \lim_{k \to \infty} \sigma_T^k(1_H) = 0,$$

where $\sigma_T : L(H) \to L(H)$ is the linear map defined by $\sigma_T(X) = \sum_{i=1}^n T_i X T_i^*$.

Let $D_T = (1_H - T^*T)^{1/2} \in L(H^n)$, $D_{T^*} = (1_H - TT^*)^{1/2} \in L(H)$ be the defect operators of T and let $\mathcal{D}_T = \overline{D_T H^n}$, $\mathcal{D}_{T^*} = \overline{D_{T^*} H}$ be the defect spaces of T, where $T \in L(H^n, H)$ is regarded as a row operator and $T^* \in L(H, H^n)$ as a column operator. One way to prove the dilation result mentioned above in the case of a pure row contraction $T \in L(H)^n$ is to show that the characteristic function $\theta_T : \mathbb{B} \to L(\mathcal{D}_T, \mathcal{D}_{T^*})$,

$$\theta_T(z) = -T + D_{T^*}(1_H - ZT^*) Z D_T$$

defines a partially isometric multiplier from $H(\mathbb{B}, \mathcal{D}_T)$ to $H(\mathbb{B}, \mathcal{D}_{T^*})$ and that T is unitarily equivalent to the compression of $M_z \in L(H(\mathbb{B}, \mathcal{D}_{T^*}))^n$ to the co-invariant subspace

$$\mathbb{H}_T = H(\mathbb{B}, \mathcal{D}_{T^*}) \ominus \theta_T H(\mathbb{B}, \mathcal{D}_T).$$

Let $m > 0$ be a positive integer and let $H_m(\mathbb{B}, \mathcal{D})$ be the $\mathcal{D}$-valued functional Hilbert space given by the reproducing kernel

$$K_m : \mathbb{B} \times \mathbb{B} \to L(\mathcal{D}), (z, w) \mapsto \frac{1_{\mathcal{D}}}{(1 - \langle z, w \rangle)^m}.$$

Then the corresponding multiplication tuple $M_z \in L(H_m(\mathbb{B}, \mathcal{D}))^n$ plays the role of a model tuple for a class of commuting Hilbert-space tuples $T \in L(H)^n$ satisfying suitable higher order positivity conditions. To be more precise, the tuple T is a row contraction if and only if $(1 - \sigma_T)(1_H) \geq 0$. A commuting tuple $T \in L(H)^n$ is called a row-m-hypercontraction, or simply an m-hypercontraction, if