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Renormalization analysis for degenerate ground states

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ABSTRACT

We consider a Hamilton operator which describes a finite dimensional quantum mechanical system with degenerate eigenvalues coupled to a field of relativistic bosons. We show that the ground state projection and the ground state energy are analytic functions of the coupling constant in a cone with apex at the origin, provided a mild infrared assumption holds. To show the result operator theoretic renormalization is used and extended to degenerate situations.

1. Introduction

Models of quantum field theory which describe low energy phenomena of quantum mechanical matter interacting with a quantized field of massless particles have been mathematically intensively investigated (see for example [27] and references therein, for an early work see [14]). These models are used to study non relativistic matter interacting with the quantized radiation field or electrons in a solid interacting with a field of phonons. Physical properties such as existence of ground states, dispersion relations,

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and resonances have been treated mathematically rigorous. In particular, the method of operator theoretic renormalization, introduced by Bach Fröhlich and Sigal [7,8], has been used in the literature to study ground states and resonances [3,4,11–13,16,20–22,25]. However, the application of operator theoretic renormalization usually requires that the unperturbed eigenstate is non degenerate or at least protected by a symmetry. In this paper we extend operator theoretic renormalization to situations where the unperturbed eigenvalue is degenerate and the degeneracy is lifted after the interaction is turned on. We note that degenerate situations do occur in physically realistic models, see for example [1]. To keep notation simple we treat the ground state. Resonances can be treated by the same ideas as used in this paper with additional notational complexity. This is planned to be addressed in a forthcoming paper.

More precisely, we consider a quantum mechanical atomic system described by a Hamilton operator acting on a so called atomic Hilbert space. For simplicity we assume that the atomic Hilbert space is finite dimensional (we expect that this assumption is not essential and can be relaxed in a straightforward way). Furthermore, we assume that the atomic system interacts with a quantized field of massless bosons by means of a linear coupling. The resulting Hamiltonian describing the total system is also referred to as generalized spin boson Hamiltonian. We assume that the interaction satisfies a mild infrared condition. The infrared condition is needed for the renormalization analysis to converge. It can be shown to include realistic models of non relativistic quantum electrodynamics by means of a so called generalized Pauli Fierz transformation [25]. We assume that the Hamiltonian of the atomic subsystem has a degenerate ground state, which is lifted by formal second order perturbation theory in the coupling constant (first order perturbation theory does not affect the ground state energy for models which we consider). We show that the ground state exists for small values of the coupling constant, a result already known in the literature [15,17,23,26]. Furthermore, we show that the ground state projection as well as the ground state energy are analytic as a function of the coupling constant in an open cone with apex at the origin. This result is new and it is in contrast to non degenerate situations, where it has been shown that the ground state projection and the ground state energy are analytic functions of the coupling constant [16]. We do not assume that this is an artefact of our proof. In fact, we conjecture that in the degenerate case there may be situations in which the ground state projection and possibly the ground state energy are not analytic in a neighborhood of zero. In a related model, where a hydrogen atom is minimally coupled to the quantized electromagnetic field, non analyticity in the fine structure constant has been shown [9].

Although we do not obtain analyticity in a neighborhood of zero, analyticity in a cone is of interest in its own right. It is for example a necessary ingredient to show Borel summability. Borel summability methods allow to recover a function from its asymptotic expansion. An asymptotic expansion may for example be obtained using the techniques employed in [2,5,6,10,18].

In the following section we state the model and the main result. The subsequent sections are devoted to the proof of the main result.

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