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POSITIVE CLUSTERS FOR SMOOTH PERTURBATIONS OF A CRITICAL ELLIPTIC EQUATION IN DIMENSIONS FOUR AND FIVE

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ABSTRACT. We construct clustering positive solutions for a perturbed critical elliptic equation on a closed manifold of dimension $n = 4, 5$. Such a construction is already available in the literature in dimensions $n \geq 6$ (see for instance [10, 14, 30, 32, 36]) and not possible in dimension 3 by [27]. This also provides new patterns for the Lin–Ni [23] problem on closed manifolds and completes results by Brézis and Li [8] about this problem.

1. INTRODUCTION AND MAIN RESULT

Let (M^n, g) be a smooth closed Riemannian manifold of dimension $n \geq 3$, and $2^\star = \frac{2n}{n-2}$ be the critical Sobolev exponent for the embeddings of $H^1(M)$ into the Lebesgue spaces. Given smooth perturbations $(h_\varepsilon)_\varepsilon$ of a function h_0 in M, the asymptotic behavior of a sequence $(u_\varepsilon)_\varepsilon$ of smooth positive functions satisfying

$$\Delta_g u_\varepsilon + h_\varepsilon u_\varepsilon = u_\varepsilon^{2^\star - 1} \tag{1.1}$$

for all $\varepsilon > 0$ has been intensively studied in the last decades. Here $\Delta_g = -\text{div}_g(\nabla \cdot)$ is the Laplace–Beltrami operator. If such a sequence $(u_\varepsilon)_\varepsilon$ is bounded in $H^1(M)$, then we know from Struwe [41] that there exist $k \in \mathbb{N}$, k sequences $(\mu_{1,\varepsilon})_\varepsilon, \dots, (\mu_{k,\varepsilon})_\varepsilon$ of positive numbers converging to 0, and k sequences $(\xi_{1,\varepsilon})_\varepsilon, \dots, (\xi_{k,\varepsilon})_\varepsilon$ of points converging to $\xi_1, \dots, \xi_k$ in M such that

$$u_\varepsilon = u_0 + \sum_{i=1}^{k} \left(\frac{\sqrt{n\,(n-2)}\mu_{i,\varepsilon}}{\mu_{i,\varepsilon}^2 + d_g(\xi_{i,\varepsilon}, \cdot)^2} \right)^{\frac{n-2}{2}} + o(1) \tag{1.2}$$

up to a subsequence, where $o(1) \to 0$ strongly and $u_\varepsilon \rightharpoonup u_0$ in $H^1(M)$ as $\varepsilon \to 0$. If the sequence $(u_\varepsilon)_\varepsilon$ is not uniformly bounded, then we say that $(u_\varepsilon)_\varepsilon$ blows up and in this case, it follows from classical elliptic estimates that k is non-zero in (1.2). If $\xi_1 = \dots = \xi_k = \xi_0$, then we say that $(u_\varepsilon)_\varepsilon$ blows up with k peaks at the point ξ_0.

In the case of dimension 3, it was proved by Li and Zhu [27] (see Theorem 6.3 in Hebey [21]) that $\xi_1, \dots, \xi_k$ are necessarily distinct in (1.2). By contrast, in the case of dimensions larger than or equal to 6, Druet

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