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Positive clusters for smooth perturbations of a critical elliptic equation in dimensions four and five

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**POSITIVE CLUSTERS FOR SMOOTH  
PERTURBATIONS OF A CRITICAL ELLIPTIC  
EQUATION IN DIMENSIONS FOUR AND FIVE**

PIERRE-DAMIEN THIZY AND JÉRÔME VÉTOIS

ABSTRACT. We construct clustering positive solutions for a perturbed critical elliptic equation on a closed manifold of dimension  $n = 4, 5$ . Such a construction is already available in the literature in dimensions  $n \geq 6$  (see for instance [10, 14, 30, 32, 36]) and not possible in dimension 3 by [27]. This also provides new patterns for the Lin–Ni [23] problem on closed manifolds and completes results by Brézis and Li [8] about this problem.

1. INTRODUCTION AND MAIN RESULT

Let  $(M^n, g)$  be a smooth closed Riemannian manifold of dimension  $n \geq 3$ , and  $2^* = \frac{2n}{n-2}$  be the critical Sobolev exponent for the embeddings of  $H^1(M)$  into the Lebesgue spaces. Given smooth perturbations  $(h_\varepsilon)_\varepsilon$  of a function  $h_0$  in  $M$ , the asymptotic behavior of a sequence  $(u_\varepsilon)_\varepsilon$  of smooth positive functions satisfying

$$\Delta_g u_\varepsilon + h_\varepsilon u_\varepsilon = u_\varepsilon^{2^*-1} \quad (1.1)$$

for all  $\varepsilon > 0$  has been intensively studied in the last decades. Here  $\Delta_g = -\operatorname{div}_g(\nabla \cdot)$  is the Laplace–Beltrami operator. If such a sequence  $(u_\varepsilon)_\varepsilon$  is bounded in  $H^1(M)$ , then we know from Struwe [41] that there exist  $k \in \mathbb{N}$ ,  $k$  sequences  $(\mu_{1,\varepsilon})_\varepsilon, \dots, (\mu_{k,\varepsilon})_\varepsilon$  of positive numbers converging to 0, and  $k$  sequences  $(\xi_{1,\varepsilon})_\varepsilon, \dots, (\xi_{k,\varepsilon})_\varepsilon$  of points converging to  $\xi_1, \dots, \xi_k$  in  $M$  such that

$$u_\varepsilon = u_0 + \sum_{i=1}^k \left( \frac{\sqrt{n(n-2)}\mu_{i,\varepsilon}}{\mu_{i,\varepsilon}^2 + d_g(\xi_{i,\varepsilon}, \cdot)^2} \right)^{\frac{n-2}{2}} + o(1) \quad (1.2)$$

up to a subsequence, where  $o(1) \rightarrow 0$  strongly and  $u_\varepsilon \rightharpoonup u_0$  in  $H^1(M)$  as  $\varepsilon \rightarrow 0$ . If the sequence  $(u_\varepsilon)_\varepsilon$  is not uniformly bounded, then we say that  $(u_\varepsilon)_\varepsilon$  blows up and in this case, it follows from classical elliptic estimates that  $k$  is non-zero in (1.2). If  $\xi_1 = \dots = \xi_k = \xi_0$ , then we say that  $(u_\varepsilon)_\varepsilon$  blows up with  $k$  peaks at the point  $\xi_0$ .

In the case of dimension 3, it was proved by Li and Zhu [27] (see Theorem 6.3 in Hebey [21]) that  $\xi_1, \dots, \xi_k$  are necessarily distinct in (1.2). By contrast, in the case of dimensions larger than or equal to 6, Druet

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