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Many-body localization in the droplet spectrum of the random XXZ quantum spin chain

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A B S T R A C T

We study many-body localization properties of the disordered XXZ spin chain in the Ising phase. Disorder is introduced via a random magnetic field in the z-direction. We prove a strong form of dynamical exponential clustering for eigenstates in the droplet spectrum: For any pair of local observables separated by a distance ℓ, the sum of the associated correlators over these states decays exponentially in ℓ, in expectation. This exponential clustering persists under the time evolution in the droplet spectrum. Our result applies to the large disorder regime as well as to the strong Ising phase at fixed disorder, with bounds independent of the support of the observables.

Contents

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1. Introduction

1.1. Many-body localization

Understanding the structure and complexity of the eigenstates of “typical” local Hamiltonians is one of the central problems in Condensed Matter Physics and Quantum Complexity Theory. The concept of disorder induced localization has been introduced in the seminal work of Anderson [6], who suggested a mechanism responsible for the absence of diffusion of waves in disordered media. This mechanism is well understood by now in the single particle case, both physically and mathematically. In random Schrödinger operators, localization manifests itself as pure point spectrum, with the corresponding eigenvectors exhibiting exponentially fast spatial decay, for almost all configurations of the environment.

It turns out that many manifestations of single-particle Anderson localization remain valid if one considers a fixed number of interacting particles, e.g., [14,5,36]. The methodology used in these works is unfortunately inadequate to study the thermodynamic limit of an electron gas in a random environment, i.e., an infinite volume limit in which the number of electrons grows proportionally to the volume. In the landmark paper [8] it was suggested that some hallmarks of localization indeed survive the passage to a true many-body system. This has sparked extensive efforts in the physics community to understand this phenomenon, known as many-body localization (MBL), see, e.g., [7,9,12,47,50,53,54]. This compilation, far from being comprehensive, lists only a few salient works closely related to the current paper. Definitions and heuristic arguments that lay a general foundation for an analytical approach to MBL were introduced in [27], based on earlier work on gapped systems (e.g., [26]), by identifying their possible counterparts in mobility gapped systems.

Investigating the combined effects of disorder and interactions on large quantum systems is difficult, because the many-body quantum states are extremely complicated objects. Even an approximate description of such a state, in general, requires the introduction of an exponential number of parameters (in terms of the system’s size). As a consequence, the basic questions related to the behavior of disordered many-body quantum systems are still subject of debate in the physics community. However, a clearer phenomenological picture has by now been drawn in the so called *fully many-body localized* regime, e.g., [13,29,30,51].

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