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Dynamics and spectra of composition operators on the Schwartz space ☆

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ABSTRACT

In this paper we study the dynamics of the composition operators defined in the Schwartz space $\mathcal{S}(\mathbb{R})$ of rapidly decreasing functions. We prove that such an operator is never supercyclic and, for monotonic symbols, it is power bounded only in trivial cases. For a polynomial symbol φ of degree greater than one we show that the operator is mean ergodic if and only if it is power bounded and this is the case when φ has even degree and lacks fixed points. We also discuss the spectrum of composition operators.

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1. Introduction and notation

We study the dynamics of composition operators defined in the Schwartz space $\mathcal{S}(\mathbb{R})$ of smooth rapidly decreasing functions. The smooth functions $\varphi : \mathbb{R} \to \mathbb{R}$ for which the composition operator $C_\varphi : \mathcal{S}(\mathbb{R}) \to \mathcal{S}(\mathbb{R}), f \mapsto f \circ \varphi$, is well defined were characterized by the second and the third author in [14], where the compactness and closed range of the operator is analyzed. In this paper we discuss the behavior of the orbits $\{C_\varphi^n(f) : n \in \mathbb{N}\}$. Dynamics of composition operators in Banach spaces of analytic functions on the unit disc have been broadly investigated. There are a lot of results relating the dynamics of C_φ to that of φ [12,26]. In the last years composition operators on spaces of smooth functions on the reals have attracted the attention of several authors. Dynamics on the space of real analytic functions is analyzed by Bonet and Domański in [8,9]. More recently, Kennesey, Wengenroth and Przestacki have investigated composition operators on the space $C^\infty(\mathbb{R})$ of smooth functions on $\mathbb{R}$ (see [20,22–24]). The dynamics of composition operators on $C^\infty(\mathbb{R})$ has been studied in [25].

In [16] it is proved that $\mathcal{S}(\mathbb{R})$ admits continuous linear operators for which every nonzero vector is hypercyclic. In Section 2 we prove that composition operators on the Schwartz class cannot provide these kind of examples, since they are neither hypercyclic nor supercyclic. We recall that an operator T on a locally convex space (lcs) is said to be hypercyclic if there exists a dense orbit $O(T, x) = \{T^n(x) : n \in \mathbb{N}\}$. The operator is supercyclic if there exists $x \in X$ such that the projective orbit $\mathbb{K}O(T, x) = \{\lambda T^n(x) : \lambda \in \mathbb{K}, n \in \mathbb{N}\}$ is dense. It follows from the definition that only separable spaces support supercyclic operators. There is a vast literature studying hypercyclicity and supercyclicity in concrete operators defined on Banach or Fréchet spaces (see the monographies [3,19]).

An operator $T : X \to X$ is said to be power bounded if $\{T^n : n \in \mathbb{N}\}$ is an equicontinuous set. If X is a Fréchet space then T is power bounded if and only if $\{T^n(x) : n \in \mathbb{N}\}$ is bounded for each $x \in X$. A closely related concept to power boundedness is that of *mean ergodicity*. Given $T \in L(X)$, the Cesàro means of T are defined as $T_{[n]} = \sum_{k=1}^n T^k/n$. T is said to be mean ergodic when $T_{[n]}$ converges to an operator P, which is always a projection, in the strong operator topology, i.e. if $(T_{[n]}(x))_n$ is convergent to $P(x)$ for each $x \in X$. Clearly, if T is mean ergodic then $\lim_{n\to\infty} \frac{T^n(x)}{n} = 0$ for each $x \in E$. The operator is called *uniformly mean ergodic* if this convergence happens uniformly on bounded sets, that is $(T_{[n]})_n$ is convergent to P in $L_b(X)$. When X is a Banach space this means that the convergence happens in the operator norm topology. If X is reflexive then each power bounded operator is mean ergodic. The result was proved by Lorch [21] for reflexive Banach spaces extending the classical Von Neumann mean ergodic theorem valid for unitary operators defined on a Hilbert space. Albanese, Bonet and Ricker [1] showed that the result remains true if X is a Fréchet space. Moreover, if X is *Montel*, i.e. barrelled and such that closed and bounded subsets are compact, it follows that mean ergodicity and uniform mean ergodicity are equivalent concepts. Power boundedness and mean ergodicity are mainly studied as a theoretical tool for analyzing the structure of

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