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Crossed-products extensions, of L_p-bounds for amenable actions

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A R T I C L E I N F O

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A B S T R A C T

We will extend earlier transference results due to Neuwirth and Ricard from the context of noncommutative L_p-spaces associated with amenable groups to that of noncommutative L_p-spaces associated with crossed-products of amenable actions. Namely, if $m : G \to \mathbb{C}$ is a completely bounded Fourier multiplier on L_p, then it extends to the crossed-product with similar bounds provided that the action θ is amenable and trace-preserving. Furthermore, our construction also allows to extend G-equivariant completely bounded operators acting on the space part to the crossed-product provided that the generalized Følner sets of the action θ satisfy certain accretivity property. As a corollary we obtain stability results for maximal L_p-bounds over crossed products. We derive, using that stability results, an application to the boundedness of smooth multipliers in the L_p-spaces of group algebras.

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0. Introduction

The purpose of this article is to study transference results for operators acting on the noncommutative L_p-spaces of crossed products. Let G be a group and let $\mathcal{L}G$ be its (left) regular von Neumann algebra. If $m : G \to \mathbb{C}$ is a function, then its associated Fourier multiplier T_m is the potentially unbounded operator on $\mathcal{L}G$ given by linear extension of $\lambda_g \mapsto m(g)\,\lambda_g$. Similarly, its associated Herz–Schur multiplier M_m is the potentially unbounded operator on $\mathcal{B}(L_2 G)$ given by $e_{g\,h} \mapsto m(g^{-1}h)e_{g\,h}$, where $e_{g\,h}$ are the matrix units. The boundedness or complete boundedness of such operators have been throughly studied, specially in connection with approximation properties of groups and operator algebras. The boundedness over the L_p-spaces of multiplier operators have received attention in recent years both because of its connection with approximation properties, see [27], and as part of a program to extend harmonic analysis to the setting of noncommutative mathematics. In that regard, both Hömander–Mikhlin theorems, see [22,15], and boundedness results for Riesz transforms, see [23], have been obtained.

Fourier and Schur multipliers are closely related objects. Bożejko and Fendler proved in [5] that M_m is bounded over $\mathcal{B}(\ell_2 G)$ iff T_m is completely bounded over $\mathcal{L}G$. Ir is also well known that the complete boundedness of M_m is implied by its boundedness. Indeed, such result is a particular case of a larger family of automatic continuity theorems for bimodular maps, see [40]. The complete boundedness of T_m over $\mathcal{L}G$ is long known to be strictly stronger of the boundedness, see for example [11]. It is unknown whether the boundedness over L_p spaces of M_m and T_m are equivalent and whether there are L_p-bounded multipliers which are not completely bounded. Regarding the first question, a partial result was obtained by Neuwirth and Ricard. They proved that for every $1 \leq p \leq \infty$ and discrete group G the complete boundedness, of $T_m : L_p(\mathcal{L}G) \to L_p(\mathcal{L}G)$ is *a* priori stronger than that of $M_m : S_p(\ell_2 G) \to S_p(\ell_2 G)$. When the group G is amenable they showed that both notions are equivalent. Their results were later extended to general locally compact groups in [7]. The treatment of the general, locally compact, case requires a leap in the level of technicality due to the fact that, when G is not unimodular, the associated L_p-spaces have to be defined using a non-tracial weight, which made necessary the use of either modular theory or spatial theory of von Neumann algebras, see [42] or [20]. The technique used in [29] to treat the amenable case was to use the Følner condition to construct a net of maps from $L_p(\mathcal{L}G)$ to $S_p(\ell_2 G)$ that behave asymptotically like an isometry and that intertwine Fourier and Herz–Schur multipliers. The main technical tool of this article will be an extension of such asymptotic intertwining technique from the context of amenable groups to that of *amenable actions*, see [6, Section 4.3] or [49, Chapter 4] for a precise definition.

Let $\mathcal{M}$ be a semifinite von Neumann algebra with a tracial weight, $\theta : G \to \mathrm{Aut}(\mathcal{M})$ a trace-preserving action and $\mathcal{M} \rtimes_\theta G$ its crossed product. The crossed product can be endowed with a natural normal and faithful weight that extends the weight of $\mathcal{M}$. That allows us to define the L_p-spaces over $\mathcal{M} \rtimes_\theta G$ in a natural way. Our main technical result will be that if θ is an amenable action we can construct a net of maps $j_p^\alpha : L_p(\mathcal{M} \rtimes_\theta G) \to$