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Hypercyclic algebras for convolution and composition operators [☆]

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A R T I C L E I N F O

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A B S T R A C T

We provide an alternative proof to those by Shkarin and by Bayart and Matheron that the operator D of complex differentiation supports a hypercyclic algebra on the space of entire functions. In particular we obtain hypercyclic algebras for many convolution operators not induced by polynomials, such as $\cos(D)$, De^D, or $e^D - aI$, where $0 < a \leq 1$. In contrast, weighted composition operators on function algebras of analytic functions on a plane domain fail to support supercyclic algebras.

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1. Introduction

A special task in linear dynamics is to understand the algebraic and topological properties of the set

$$HC(T) = \{f \in X : \quad \{f, Tf, T^2 f, \dots\} \text{ is dense in } X\}$$

of hypercyclic vectors for a given operator T on a topological vector space X. It is well known that in general $HC(T)$ is always connected and is either empty or contains a dense infinite-dimensional linear subspace (but the origin), see [26]. Moreover, when $HC(T)$ is non-empty it sometimes contains (but zero) a closed and infinite dimensional linear subspace, but not always [8,19]; see also [7, Ch. 8] and [21, Ch. 10].

When X is a topological algebra it is natural to ask whether $HC(T)$ can contain, or must always contain, a subalgebra (but zero) whenever it is non-empty; any such subalgebra is said to be a *hypercyclic algebra* for the operator T. Both questions have been answered by considering convolution operators on the space $X = H(\mathbb{C})$ of entire functions on the complex plane $\mathbb{C}$, endowed with the compact-open topology; that convolution operators (other than scalar multiples of the identity) are hypercyclic was established by Godefroy and Shapiro [18], see also [14,22,2], together with the fact that convolution operators on $H(\mathbb{C})$ are precisely those of the form

$$f \overset{\Phi(D)}{\mapsto} \sum_{n=0}^{\infty} a_n D^n f \quad (f \in H(\mathbb{C}))$$

where $\Phi(z) = \sum_{n=0}^{\infty} a_n z^n \in H(\mathbb{C})$ is of (growth order one and finite) exponential type (i.e., $|a_n| \leq M \ \frac{R^n}{n!} (n = 0, 1, \dots)$, for some $M, R > 0$) and where D is the operator of complex differentiation. Aron et al. [4,5] showed that no translation operator τ_a on $H(\mathbb{C})$

$$\tau_a(f)(z) = f(z + a) \quad f \in H(\mathbb{C}), z \in \mathbb{C}$$

can support a hypercyclic algebra, in a strong way:

Theorem 1. (Aron, Conejero, Peris, Seoane) *For each integer $p > 1$ and each $f \in HC(\tau_a)$, the non-constant elements of the orbit of f^p under τ_a are those entire functions for which the multiplicities of their zeros are integer multiples of p.*

In sharp contrast with the translations operators, they also showed that the collection of entire functions f for which every power f^n $(n = 1, 2, \dots)$ is hypercyclic for D is residual in $H(\mathbb{C})$. Later Shkarin [25, Thm. 4.1] showed that $HC(D)$ contained both a hypercyclic subspace and a hypercyclic algebra, and with a different approach Bayart and Matheron [7, Thm. 8.26] also showed that the set of $f \in H(\mathbb{C})$ that generate an algebra consisting entirely (but the origin) of hypercyclic vectors for D is residual in $H(\mathbb{C})$, and by using the latter approach we now know the following:

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