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Threshold singularities of the spectral shift function for a half-plane magnetic Hamiltonian

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A B S T R A C T

We consider the Schrödinger operator with constant magnetic field defined on the half-plane with a Dirichlet boundary condition, H_0, and a decaying electric perturbation V. We study the Spectral Shift Function (SSF) associated to the pair $(H_0 + V, H_0)$ near the Landau levels, which are thresholds in the spectrum of H_0. For perturbations of a fixed sign, we estimate the SSF in terms of the eigenvalue counting function for certain compact operators. If the decay of V is power-like, then using pseudodifferential analysis, we deduce that there are singularities at the thresholds and we obtain the corresponding asymptotic behavior of the SSF. Our technique gives also results for the Neumann boundary condition.

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1. Introduction

1.1. Motivation

We consider H_0^D (resp., H_0^N), the Dirichlet (resp., Neumann) self-adjoint realization of the magnetic Schrödinger operator

$$-\frac{\partial^2}{\partial x^2} + \left(-i\frac{\partial}{\partial y} - bx\right)^2, \quad b > 0, \tag{1.1}$$

in the half-plane $\mathbb{R}_+ \times \mathbb{R}$ ($\mathbb{R}_+ := (0, \infty)$).

Our goal is to analyze the effects on the spectrum when a relatively compact perturbation of H_0^D or H_0^N is introduced. The perturbations under consideration will be real electric potentials V that decay to zero at infinity in $\mathbb{R}_+ \times \mathbb{R}$.

Such effect is now well understood for perturbations of the so-called *Landau Hamiltonian* H_L i.e., the magnetic Schrödinger operator (1.1) but defined for the whole plane $\mathbb{R}^2$. The Landau Hamiltonian has pure point spectrum with eigenvalues of infinite multiplicity (the so called *Landau levels* $\mathcal{E}_j$, $j \in \mathbb{N}$). It is established that perturbations by a decaying electric potential of a definite sign, even if it is compactly supported, produce an accumulation of discrete eigenvalues around the Landau levels (see [34,22,37,26,14,39]). Using variational methods, it can be seen that the distribution of these eigenvalues is governed by the counting function of the eigenvalues of the compact Toeplitz operators P_jVP_j, where P_j is the spectral projection onto $\mathrm{Ker}(H_L - \mathcal{E}_j)$. Then, depending on the decay rate of V, it is possible to obtain the asymptotic behavior of the counting functions of the eigenvalues of $H_L + V$, near each Landau Level. Tools from pseudodifferential analysis, together with variational and Tauberian methods have been used to obtain this behavior for V: power-like decaying [34,22], exponentially decaying [37], compactly supported [37,26,14,33,38]. Also magnetic ([39]) and geometric perturbations ([32,29,17]) of H_L have been considered.

In our case, on the half-plane, the spectrum of H_0^D (resp. H_0^N) is rather different from that of H_L. It is purely absolutely continuous, given by $\sigma(H_0^D) = [b, \infty)$ (resp. $\sigma(H_0^N) = [\Theta_0, \infty)$, $0 < \Theta_0 < b$). From a dynamical point of view, this difference is related to the fact that in $\mathbb{R}^2$ the classical trajectories are circles, while in $\mathbb{R}_+ \times \mathbb{R}$ there exist propagation phenomena along the boundary $\{0\} \times \mathbb{R}$. The accumulation of the discrete spectrum of $H_0^D + V$ and of $H_0^N + V$, for compactly supported potentials V, was studied in [8]. However, to our best knowledge, there are no results concerning the continuous spectrum.

A natural tool to extend the eigenvalue counting function, from the discrete spectrum into the continuous spectrum, is the spectral shift function (SSF) (see (1.6) and (2.11) below). For example, the SSF is studied for the Schrödinger operator with constant magnetic field in $\mathbb{R}^3$, which has purely absolutely continuous spectrum, and it is proved that this function admits singularities at the Landau levels (see [13,5] and other magnetic

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