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SUMMABILITY PROPERTIES OF GABOR EXPANSIONS

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Abstract. We show that there exist complete and minimal systems of time-frequency shifts of Gaussians in $L^2(\mathbb{R})$ which are not strong Markushevich basis (do not admit the spectral synthesis). In particular, it implies that there is no linear summation method for general Gaussian Gabor expansions. On the other hand we prove that the spectral synthesis for such Gabor systems holds up to one dimensional defect.

1. Introduction and the main results

Gabor analysis is an important part of the modern time-frequency analysis. It deals with the expansion of functions in $L^2(\mathbb{R}^n)$ in the series in the time frequency shifts of a given "window" φ,

$$\tau_{x,y}\varphi(s) := e^{2\pi i<y,s>}\varphi(s-x), \qquad (x,y)\in\Lambda,$$

Λ being a discrete subset of $\mathbb{R}^n\times\mathbb{R}^n$.

For the (most frequently used) Gaussian window $\gamma(s)=e^{-\pi s^2}$, these expansions are closely related to the corresponding uniqueness, sampling and interpolation problems in the Fock space. The case $n=1$ corresponds here to the classical Fock space of one complex variable. To study expansions of $f\in L^2(\mathbb{R}^n)$ into the series with the respect to the system

$$\mathcal{G}_\Lambda(\varphi) := \{\tau_{x,y}\varphi\}_{(x,y)\in\Lambda}$$

we need, first of all, the completeness property. On the other hand, for these expansions to be unique we should require the minimality property of the system $\mathcal{G}_\Lambda$. In 1946 Gabor considered the system

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