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Spectral theory of one-channel operators and application to absolutely continuous spectrum for Anderson type models

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A B S T R A C T

A one-channel operator is a self-adjoint operator on $\ell^2(\mathbb{G})$ for some countable set $\mathbb{G}$ with a rank 1 transition structure along the sets of a quasi-spherical partition of $\mathbb{G}$. Jacobi operators are a very special case. In essence, there is only one channel through which waves can travel across the shells to infinity. This channel can be described with transfer matrices which include scattering terms within the shells and connections to neighboring shells. Not all of the transfer matrices are defined for some countable set of energies. Still, many theorems from the world of Jacobi operators are translated to this setup. The results are then used to show absolutely continuous spectrum for the Anderson model on certain finite dimensional graphs with a one-channel structure. This result generalizes some previously obtained results on antitrees.

1. Introduction and results

The main purpose of this paper is to discuss the probably most general frame work of Hermitian operators which allow a description through 2×2 transfer matrices. We

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will call these 'one-channel operators'. In terms of classifying discrete Hermitian operators and their spectral theory we find this an interesting class. Despite the one-channel structure, the underlying graph-structures can have any kind of growth and are not necessarily one-dimensional. The work is inspired from [16], but the considered operators are more general. Part of the motivation for this work comes from the theory of random disordered systems as considered in the Mathematical Physics community. There has quite been some interest in discrete (random) Schrödinger operators associated to certain discrete graph structures, e.g. [1,2,4,5,7,8,10,11,13]. In one-dimensional systems (Jacobi operators) transfer matrices are the main tool to study such operators and their behavior can be connected to the spectrum and the spectral type in various ways [3,6,9,12]. Due to transfer matrices, a local solution to the eigenvalue equation imposes a (formal) global one and a local analysis can lead to a full understanding of the spectral properties. Here we generalize some of these methods to a more general class.

For general Schrödinger type operators on general graphs, the situation is typically much more difficult. This is part of the reason that the type of the bulk spectrum of the Anderson model in $\mathbb{Z}^d$ at small disorder is not understood. It is a main open conjecture that for $d = 2$ it should be pure point and for $d \geq 3$ there should be some absolutely continuous spectrum (almost surely). Schrödinger operators associated to locally finite graph structures with hopping only along edges can always be brought into a block tri-diagonal matrix form by spherical-type partitions of the graph, cf. [4]. However, the block sizes vary and typically grow. One still has resolvent identities, but generally the analysis can get very complicated.

As an application of this work we obtain absolutely continuous spectrum for Anderson models on families of graphs of finite dimensional growth rate with dimension $d > 2$. We call these graphs partial antitrees as they partially have the structure as the antitrees in [16]. This is a first step of improving the ac-spectrum result in [16] towards more general classes of graphs with finite dimensional growth.

Previously to the work in [16] absolutely continuous spectrum for Anderson models with independent identically distributed potential had only been shown on infinite dimensional trees and tree-like graph structures [10,1,5,11,7,2,14,15]. All these graph structures significantly simplify the general local Green's function relations as in [4] and one has some simple local transfer mechanism to study them. There has also been some work to approach the expected localization in two dimensions by certain graphs which in some sense are in between one and two dimensions [13,16].

It is our agenda to further study certain hybrid graphs of lattice structures and antitrees and analyze what is happening with the absolutely continuous part of the spectrum for the Anderson models on these graphs. We hope that this study can help to eventually make some progress in analyzing the important $\mathbb{Z}^d$-Anderson model towards showing existence of absolutely continuous spectrum.

Readers from the Mathematical Physics community should be aware that we will use a more mathematics-like notation in this paper: for instance λ will be used as the spectral

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