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BEURLING DIMENSION AND SELF-SIMILAR MEASURES

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ABSTRACT. In this paper the authors study the Beurling dimension of Bessel sets and frame spectra of some self-similar measures on \mathbb{R}^d and obtain their exact upper bound of the dimensions, which is the same given by Dutkay et.al (Adv. Math. **226** (2011), 285-297). The upper bound is attained in usual cases and some examples are given to explain our theory.

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1. INTRODUCTION

Let μ be a Borel probability measure with compact support in \mathbb{R}^d . We call μ a *Fourier-Bessel measure* with a *Bessel set or Bessel sequence* Λ in \mathbb{R}^d if

$$\sum_{\lambda \in \Lambda} |\langle f, e_\lambda \rangle|^2 \leq B \|f\|^2, \quad \forall f \in L^2(\mu),$$

where $e_\lambda = e^{-2\pi i \langle \lambda, x \rangle}$, $\langle x, y \rangle$ is the standard inner product in \mathbb{R}^d and B is a *Bessel bound*. Moreover, if in addition there exists $A > 0$ such that

$$A \|f\|^2 \leq \sum_{\lambda \in \Lambda} |\langle f, e_\lambda \rangle|^2 \leq B \|f\|^2, \quad \forall f \in L^2(\mu).$$

Then μ is called a (*Fourier*) *frame spectral measure* with a *frame spectrum* Λ , and A, B are called the lower and upper frame bounds respectively. In particular, μ is called a *Riesz*

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