Accepted Manuscript

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PII: S0022-1236(17)30325-7
DOI: http://dx.doi.org/10.1016/j.jfa.2017.08.011
Reference: YJFAN 7857

To appear in: *Journal of Functional Analysis*

Received date: 6 June 2017
Accepted date: 10 August 2017

Please cite this article in press as: X.-G. He et al., Beurling dimension and self-similar measures, *J. Funct. Anal.* (2017), http://dx.doi.org/10.1016/j.jfa.2017.08.011

BEURLING DIMENSION AND SELF-SIMILAR MEASURES

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ABSTRACT. In this paper the authors study the Beurling dimension of Bessel sets and frame spectra of some self-similar measures on $\mathbb{R}^d$ and obtain their exact upper bound of the dimensions, which is the same given by Dutkay et.al (Adv. Math. **226** (2011), 285-297). The upper bound is attained in usual cases and some examples are given to explain our theory.

CONTENTS

1. INTRODUCTION

Let μ be a Borel probability measure with compact support in $\mathbb{R}^d$. We call μ a *Fourier-Bessel measure* with a *Bessel set or Bessel sequence* Λ in $\mathbb{R}^d$ if

$$\sum_{\lambda\in\Lambda} |\langle f,\ e_\lambda\rangle|^2 \le B\|f\|^2, \qquad \forall f \in L^2(\mu),$$

where $e_\lambda = e^{-2\pi i\langle\lambda,\ x\rangle}$, $\langle x,\ y\rangle$ is the standard inner product in $\mathbb{R}^d$ and B is a *Bessel bound*. Moreover, if in addition there exists $A > 0$ such that

$$A\|f\|^2 \le \sum_{\lambda\in\Lambda} |\langle f,\ e_\lambda\rangle|^2 \le B\|f\|^2, \qquad \forall f \in L^2(\mu).$$

Then μ is called a *(Fourier) frame spectral measure* with a *frame spectrum* Λ, and A, B are called the lower and upper frame bounds respectively. In particular, μ is called a *Riesz*

1991 *Mathematics Subject Classification.* 28A80; 42C05.

Key words and phrases. Beurling dimension; Fourier-Bessel measure; Fourier-frame; Hausdorff dimension; self-similar iterated function system.

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This work was supported by the National Natural Science Foundation of China 11271148.