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Global well-posedness and long-time dynamics for a higher order quasi-geostrophic type equation

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ABSTRACT

In this paper we study a higher order viscous quasi-geostrophic type equation. This equation was derived in [11] as the limit dynamics of a singularly perturbed Navier–Stokes–Korteweg system with Coriolis force, when the Mach, Rossby and Weber numbers go to zero at the same rate.

The scope of the present paper is twofold. First of all, we investigate well-posedness of such a model on the whole space $\mathbb{R}^2$: we prove that it is well-posed in H^s for any $s \geq 3$, globally in time. Interestingly enough, we show that this equation owns two levels of energy estimates, for which one gets existence and uniqueness of weak solutions with different regularities (namely, H^3 and H^4 regularities); this fact can be viewed as a remainder of the so called BD-entropy structure of the original system.

In the second part of the paper we investigate the long-time behavior of these solutions. We show that they converge to the solution of the corresponding linear parabolic type equation, with same initial datum and external force. Our proof is based on dispersive estimates both for the solutions to the linear and non-linear problems.

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1. Introduction

In this paper we are concerned with well-posedness and long-time dynamics issues for the non-linear parabolic-type equation

$$\partial_t \left(\mathrm{Id} - \Delta + \Delta^2\right) r \,+\, \nabla^\perp(\mathrm{Id} - \Delta) r \cdot \nabla \Delta^2 r \,+\, \mu\, \Delta^2(\mathrm{Id} - \Delta) r \,=\, f\,, \tag{1}$$

where r and f are functions of $(t,x) \in \mathbb{R}_+ \times \mathbb{R}^2$. The parameter $\mu > 0$ will be kept fixed throughout all the paper; it can be interpreted as a sort of viscosity coefficient. We supplement equation (1) with the initial condition $r_{|t=0} = r_0$, where r_0 is a suitably smooth function defined over $\mathbb{R}^2$.

1.1. Derivation of the model

Equation (1) was derived in [11] as the equation describing the limit dynamics, for $\varepsilon \to 0$, of the following singular perturbation problem:

$$\begin{cases} \partial_t \rho_\varepsilon + \mathrm{div}\,(\rho_\varepsilon u_\varepsilon) \,=\, 0 \\ \partial_t\,(\rho_\varepsilon u_\varepsilon) + \mathrm{div}\,\big(\rho_\varepsilon u_\varepsilon \otimes u_\varepsilon\big) + \dfrac{1}{\varepsilon^2}\,\nabla P(\rho_\varepsilon) + \dfrac{1}{\varepsilon}\,e^3 \times \rho_\varepsilon u_\varepsilon - \nu \mathrm{div}\,\big(\rho_\varepsilon D u_\varepsilon\big) \\ \qquad - \dfrac{1}{\varepsilon^2}\,\rho_\varepsilon \nabla\Delta\rho_\varepsilon \,=\, 0\,. \end{cases} \tag{2}$$

The previous equations are posed on $\mathbb{R}^2 \times\,]0,1[$ and supplemented by complete slip boundary conditions, which allow to avoid boundary layers effects.

System (2) is the so-called Navier–Stokes–Korteweg system; it describes the dynamics of a weakly compressible viscous fluid, whose motion is mainly influenced by internal tension forces and Earth rotation. Here above, at each value of ε fixed, the scalar function $\rho_\varepsilon = \rho_\varepsilon(t,x) \geq 0$ represents the density of the fluid, $u_\varepsilon = u_\varepsilon(t,x) \in \mathbb{R}^3$ its velocity field and the function $P(\rho_\varepsilon)$ its pressure. The number $\nu > 0$ is the viscosity coefficient; the viscous stress tensor is supposed to depend on (and possibly degenerate with) the density. Finally, the term $\rho_\varepsilon\, \nabla\Delta\rho_\varepsilon$ is the capillarity tensor, which takes into account the effects of a strong surface tension, while the term $e^3 \times \rho_\varepsilon\, u_\varepsilon \,=\, \rho_\varepsilon\left(-u_\varepsilon^2, u_\varepsilon^1, 0\right)$ is the Coriolis operator, which takes into account effects due to the fast rotation of the ambient space. We refer e.g. to [5], [13] and references therein for more details on the previous model.

The scaling introduced in (2) corresponds to taking the Mach number Ma, the Rossby number Ro and the Weber number We to be all proportional to the small parameter ε. In turn, this means that we are studying the incompressible, fast rotation and strong capillarity limits at the same time, focusing our attention on their mutual interaction. See again [13] and the references therein for additional comments about the adimensionalization of the equations and for more insights on this scaling. We refer also to [12] for a related study and the derivation of a linear variable coefficients version of equation (1).

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