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Convergence rates and interior estimates in homogenization of higher order elliptic systems

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A B S T R A C T

This paper is concerned with the quantitative homogenization of $2m$-order elliptic systems with bounded measurable, rapidly oscillating periodic coefficients. We establish the sharp $O(\varepsilon)$ convergence rate in W^{m-1,p_0} with $p_0 = \frac{2d}{d-1}$ in a bounded Lipschitz domain in $\mathbb{R}^d$ as well as the uniform large-scale interior $C^{m-1,1}$ estimate. With additional smoothness assumptions, the uniform interior $C^{m-1,1}$, $W^{m,p}$ and $C^{m-1,\alpha}$ estimates are also obtained. As applications of the regularity estimates, we establish asymptotic expansions for fundamental solutions.

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1. Introduction

Let Ω be a bounded Lipschitz domain in $\mathbb{R}^d$. Consider the Dirichlet problem for a family of $2m$-order elliptic systems

$$\begin{cases} \mathcal{L}_\varepsilon u_\varepsilon = f & \text{in } \Omega, \\ Tr(D^\gamma u_\varepsilon) = g_\gamma & \text{on } \partial\Omega \quad \text{for } 0 \le |\gamma| \le m-1, \end{cases} \tag{1.1}$$

where

$$(\mathcal{L}_\varepsilon u_\varepsilon)_i = (-1)^m \sum_{|\alpha|=|\beta|=m} D^\alpha \Big(A_{ij}^{\alpha\beta}\Big(\frac{x}{\varepsilon}\Big) D^\beta u_{\varepsilon j}\Big), \quad 1 \le i,j \le n,$$

$u_{\varepsilon j}$ denotes the j-th component of the $\mathbb{R}^n$-valued function u_ε, α, β, γ are multi-indices with nonnegative integer components α_k, β_k, γ_k, $k = 1, 2, \ldots, d$, and

$$|\alpha| = \sum_{k=1}^d \alpha_k, \quad D^\alpha = D_{x_1}^{\alpha_1} D_{x_2}^{\alpha_2} \cdots D_{x_d}^{\alpha_d}.$$

We assume that the coefficients matrix $A(y) = (A_{ij}^{\alpha\beta}(y))$ is real, bounded measurable with

$$\|A_{ij}^{\alpha\beta}\|_{L^\infty(\mathbb{R}^d)} \le \frac{1}{\mu}, \tag{1.2}$$

and satisfies the coercivity condition

$$\sum_{|\alpha|=|\beta|=m} \int_{\mathbb{R}^d} D^\alpha \phi_i A_{ij}^{\alpha\beta} D^\beta \phi_j \ge \mu \sum_{|\alpha|=m} \|D^\alpha \phi\|^2_{L^2(\mathbb{R}^d)} \quad \text{for any } \phi \in C_c^\infty(\mathbb{R}^d; \mathbb{R}^n), \tag{1.3}$$

where $\mu > 0$. We also assume that

$$A_{ij}^{\alpha\beta}(y+z) = A_{ij}^{\alpha\beta}(y) \quad \text{for any } y \in \mathbb{R}^d \text{ and } z \in \mathbb{Z}^d. \tag{1.4}$$

Functions satisfying condition (1.4) will be called 1-periodic. By a linear translation, $\mathbb{Z}^d$ in (1.4) may be replaced by any lattice in $\mathbb{R}^d$.

Let $WA^{m,p}(\partial\Omega, \mathbb{R}^n)$ denote the Whitney–Sobolev space of $\dot{g} = \{g_\gamma\}_{|\gamma| \le m-1}$, which is the completion of the set of arrays of $\mathbb{R}^n$-valued functions

$$\left\{ \{D^\alpha \mathcal{G} \mid_{\partial\Omega}\}_{|\alpha| \le m-1} : \mathcal{G} \in C_c^\infty(\mathbb{R}^d; \mathbb{R}^n) \right\},$$

with respect to the norm

$$\|\dot{g}\|_{WA^{m,p}(\partial\Omega)} = \sum_{|\alpha| \le m-1} \|g_\alpha\|_{L^p(\partial\Omega)} + \sum_{|\alpha|=m-1} \|\nabla_{tan} g_\alpha\|_{L^p(\partial\Omega)}.$$

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