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Uncertainty principles for locally compact quantum groups

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ABSTRACT

In this paper, we prove the Donoho–Stark uncertainty principle for locally compact quantum groups and characterize the minimizer which are bi-shifts of group-like projections. We also prove the Hirschman–Beckner uncertainty principle for compact quantum groups and discrete quantum groups. Furthermore, we show Hardy's uncertainty principle for locally compact quantum groups in terms of bi-shifts of group-like projections.

1. Introduction

Uncertainty principles were first studied in quantum mechanics and then widely developed in harmonic analysis, information theory, and quantum information etc. In [9], Donoho and Stark proved a support-version uncertainty principle for cyclic groups and applied it in signal recovery. Later Candes, Romberg, and Tao [7] developed this un-

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certainty principle in the theory of compressed sensing. The Donoho–Stark uncertainty principle was proved for finite abelian groups [25], locally compact abelian groups[23], compact groups [1], Kac algebras [6,20]. The minimizers of the Donoho–Stark uncertainty principle for locally compact abelian groups [9,25,23] were the translations and modulations of characteristic functions of compact open subgroups. For noncommutative case, the authors [14] showed the Donoho–Stark uncertainty principle for subfactors and introduced bi-shifts of biprojections for subfactors which generalized the modulations and translations of characteristic functions of subgroups. The authors showed that the minimizers of the uncertainty principle are bi-shifts of biprojections. For infinite case, Liu and Wu [20] characterized the minimizers of the Donoho–Stark uncertainty principle for Kac algebras with biprojections.

Hirschman uncertainty principle in terms of entropies was first introduced by Hirschman in [13]. In [2], Beckner proved the uncertainty principle with sharp constant for the real line $\mathbb{R}$. The Hirschman–Beckner uncertainty principle generalized Heisenberg's uncertainty principle in quantum mechanics. This uncertainty principle was studied for locally compact abelian groups [23], Kac algebras [6,20], and subfactors [14]. The minimizers of the Hirschman–Beckner uncertainty principle were characterized in [23] and [14].

Hardy's uncertainty principle for $\mathbb{R}$ was proved in [11]. Hardy's uncertainty principles for arbitrary locally compact group were studied rarely. In [14], the authors showed that Hardy's uncertainty principle for subfactors by using the minimizers of the Donoho–Stark and the Hirschman–Beckner uncertainty principle. In [20], Liu and Wu proved Hardy's uncertainty principle for Kac algebras with biprojections. Note that the authors [14] showed that there are eight forms of a bi-shift of a biprojection and Hardy's uncertainty principle in [14,20] implies that the uniqueness of a bi-shift of a biprojection.

Locally compact quantum groups introduced by Kustermanns and Vaes [16,17] generalized locally compact groups and their duals. Compact quantum groups introduced by Woronowicz [27–30] are locally compact quantum groups. In this paper, we prove the Donoho–Stark uncertainty principle for locally compact quantum groups and characterize the minimizers of the uncertainty principle. We introduce the notion of a bi-shift of a group-like projection and show that the minimizers are bi-shifts of group-like projections. For finite abelian groups, bi-shifts of group-like projections are wave packets [10]. Wave packets are widely used in quantum mechanics, information theory, etc.

Main Theorem 1 *(Donoho–Stark uncertainty principle, Theorem 4.2, Proposition 4.7, Proposition 6.5). Suppose $\mathbb{G}$ is a locally compact quantum group. Then for any ω in $L^1(\mathbb{G}) \cap L^2(\mathbb{G})$, $1 \leq t \leq 2$, $2 \leq s \leq \infty$, we have*

$$\mathcal{S}_r(\xi_t(\omega))\mathcal{S}_r(\iota^s(\lambda(\omega))) \geq 1.$$

Moreover the following are equivalent:

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