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ON THE L^P BOUNDEDNESS OF WAVE OPERATORS FOR TWO-DIMENSIONAL SCHRÖDINGER OPERATORS WITH THRESHOLD OBSTRUCTIONS

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Abstract. Let $H = -\Delta + V$ be a Schrödinger operator on $L^2(\mathbb{R}^2)$ with real-valued potential V, and let $H_0 = -\Delta$. If V has sufficient pointwise decay, the wave operators $W_\pm = s - \lim_{t\to\pm\infty} e^{itH}e^{-itH_0}$ are known to be bounded on $L^p(\mathbb{R}^2)$ for all $1 < p < \infty$ if zero is not an eigenvalue or resonance. We show that if there is an s-wave resonance or an eigenvalue only at zero, then the wave operators are bounded on $L^p(\mathbb{R}^2)$ for $1 < p < \infty$. This result stands in contrast to results in higher dimensions, where the presence of zero energy obstructions is known to shrink the range of valid exponents p.

1. Introduction

Let $H = -\Delta+V$ be a Schrödinger operator with a real-valued potential V and $H_0 = -\Delta$. If $|V(x)| \lesssim \langle x\rangle^{-\beta}$ for some $\beta > 2$, then the spectrum of H is composed of a finite collection of non-positive eigenvalues along with the absolutely continuous spectrum on $[0,\infty)$, [24]. The wave operators are defined by the strong limits on $L^2(\mathbb{R}^n)$

$$W_\pm f = \lim_{t\to\pm\infty} e^{itH}e^{-itH_0}f. \tag{1}$$

Such limits are known to exist and be asymptotically complete for a wide class of potentials V. Furthermore, one has the identities

$$W_\pm^* W_\pm = I, \qquad W_\pm W_\pm^* = P_{ac}(H), \tag{2}$$

with $P_{ac}(H)$ the projection onto the absolutely continuous spectral subspace associated with the Schrödinger operator H.

This work continues a line of inquiry on the $L^p(\mathbb{R}^n)$ and $W^{k,p}(\mathbb{R}^n)$ boundedness of the wave operators. It is known, see [28, 29, 20, 31, 11, 3, 4, 5, 7] that the wave operators are

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