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**ON THE  $L^p$  BOUNDEDNESS OF WAVE OPERATORS FOR  
TWO-DIMENSIONAL SCHRÖDINGER OPERATORS WITH  
THRESHOLD OBSTRUCTIONS**

M. BURAK ERDOĞAN, MICHAEL GOLDBERG AND WILLIAM R. GREEN

ABSTRACT. Let  $H = -\Delta + V$  be a Schrödinger operator on  $L^2(\mathbb{R}^2)$  with real-valued potential  $V$ , and let  $H_0 = -\Delta$ . If  $V$  has sufficient pointwise decay, the wave operators  $W_{\pm} = s - \lim_{t \rightarrow \pm\infty} e^{itH} e^{-itH_0}$  are known to be bounded on  $L^p(\mathbb{R}^2)$  for all  $1 < p < \infty$  if zero is not an eigenvalue or resonance. We show that if there is an s-wave resonance or an eigenvalue only at zero, then the wave operators are bounded on  $L^p(\mathbb{R}^2)$  for  $1 < p < \infty$ . This result stands in contrast to results in higher dimensions, where the presence of zero energy obstructions is known to shrink the range of valid exponents  $p$ .

1. INTRODUCTION

Let  $H = -\Delta + V$  be a Schrödinger operator with a real-valued potential  $V$  and  $H_0 = -\Delta$ . If  $|V(x)| \lesssim \langle x \rangle^{-\beta}$  for some  $\beta > 2$ , then the spectrum of  $H$  is composed of a finite collection of non-positive eigenvalues along with the absolutely continuous spectrum on  $[0, \infty)$ , [24]. The wave operators are defined by the strong limits on  $L^2(\mathbb{R}^n)$

$$(1) \quad W_{\pm} f = \lim_{t \rightarrow \pm\infty} e^{itH} e^{-itH_0} f.$$

Such limits are known to exist and be asymptotically complete for a wide class of potentials  $V$ . Furthermore, one has the identities

$$(2) \quad W_{\pm}^* W_{\pm} = I, \quad W_{\pm} W_{\pm}^* = P_{ac}(H),$$

with  $P_{ac}(H)$  the projection onto the absolutely continuous spectral subspace associated with the Schrödinger operator  $H$ .

This work continues a line of inquiry on the  $L^p(\mathbb{R}^n)$  and  $W^{k,p}(\mathbb{R}^n)$  boundedness of the wave operators. It is known, see [28, 29, 20, 31, 11, 3, 4, 5, 7] that the wave operators are

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