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Multiplier algebras of complete Nevanlinna–Pick spaces: Dilations, boundary representations and hyperrigidity

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A B S T R A C T

We study reproducing kernel Hilbert spaces on the unit ball with the complete Nevanlinna–Pick property through the representation theory of their algebras of multipliers. We give a complete description of the representations in terms of the reproducing kernels. While representations always dilate to $*$-representations of the ambient C^*-algebra, we show that in our setting we automatically obtain coextensions. In fact, we show that in many cases, this phenomenon characterizes the complete Nevanlinna–Pick property. We also deduce operator theoretic dilation results which are in the spirit of work of Agler and several other authors. Moreover, we identify all boundary representations, compute the C^*-envelopes and determine hyperrigidity for certain analogues of the disc algebra. Finally, we extend these results to spaces of functions on homogeneous subvarieties of the ball.

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1. Introduction

In this work, we study representations of multiplier algebras of reproducing kernel Hilbert spaces with the complete Nevanlinna–Pick property. Our motivating example is the classical disc algebra $A(\mathbb{D})$, which consists of those analytic functions on the open unit disc $\mathbb{D} \subset \mathbb{C}$ which extend to continuous functions on $\overline{\mathbb{D}}$. This algebra plays a pivotal role in the theory of contractions on Hilbert space, see [47] for a classical treatment. To make this role explicit, we must consider $A(\mathbb{D})$ in relation to the Hardy space $H^2(\mathbb{D})$, which is the Hilbert space of analytic functions on $\mathbb{D}$ whose Taylor coefficients at the origin are square summable. Alternatively, the Hardy space may be described as the reproducing kernel Hilbert space on the unit disc with kernel

$$k(z,w) = \frac{1}{1 - z\overline{w}} \quad (z, w \in \mathbb{D}).$$

Then, for $\varphi \in A(\mathbb{D})$ we denote by M_φ the corresponding multiplication operator on $H^2(\mathbb{D})$. This identification allows us to regard $A(\mathbb{D})$ as an algebra of bounded linear operators on $H^2(\mathbb{D})$.

The (matrix-valued version of) von Neumann's inequality (see, for example, [41, Corollary 3.12]) shows that unital (completely) contractive representations of $A(\mathbb{D})$ are in one-to-one correspondence with contractions on a Hilbert space. Specifically, every contraction $T \in B(\mathcal{H})$ gives rise to a unital completely contractive representation

$$A(\mathbb{D}) \to B(\mathcal{H}), \quad p \mapsto p(T) \quad (p \in \mathbb{C}[z]).$$

Abstract dilation theory in the form of Arveson's dilation theorem [9] (see also [41, Corollary 7.7]) shows that this representation dilates to a $*$-representation π of $C^*(A(\mathbb{D})) \subset B(H^2(\mathbb{D}))$, in the sense that

$$p(T) = P_{\mathcal{H}} \pi(M_p)|_{\mathcal{H}} \quad (p \in \mathbb{C}[z]).$$

The C^*-algebra $C^*(A(\mathbb{D}))$ is known as the Toeplitz algebra, whose representations are well understood. Indeed, M_z is the unilateral shift, hence $\pi(M_z)$ is an isometry. The Wold decomposition therefore implies that $\pi(M_z)$ is unitarily equivalent to $M_z^{(\alpha)} \oplus U$ for some cardinal α and some unitary operator U. In particular, we see that every contraction T dilates to an operator of the form $M_z^{(\alpha)} \oplus U$.

However, a stronger statement is true. Namely, one version of Sz.-Nagy's dilation theorem [47, Theorem 4.1], combined with the Wold decomposition, asserts that every contraction T does not merely dilate, but in fact coextends to an operator of the form $M_z^{(\alpha)} \oplus U$. That is, the dilation has the additional property that $\mathcal{H}$ is invariant under $(M_z^{(\alpha)} \oplus U)^*$. In representation theoretic terms, this means that every unital completely contractive representation $\rho : A(\mathbb{D}) \to B(\mathcal{H})$ coextends to a $*$-representation π of $C^*(A(\mathbb{D}))$, in the sense that

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