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# A note on blow-up of solution for a class of semilinear pseudo-parabolic equations

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ABSTRACT

In this short note, we revisit the blow-up of solution for the initial boundary value problem of semilinear pseudo-parabolic equations with low/critical initial energy stated in Xu and Su (2013) [4], and amend the proofs of the original paper.



In this note, we reconsider the initial-boundary value of semilinear pseudo-parabolic equation

$$\begin{cases} u_t - \triangle u_t - \triangle u = u^p, & \text{in } \Omega \times (0,T), \\ u(0) = u_0, & \text{in } \Omega, \\ u = 0, & \text{on } \partial\Omega \times (0,T), \end{cases} \tag{0.1}$$

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where $u_0 \in H_0^1(\Omega)$, $T \in (0, \infty]$ and $p$ satisfies

$$\text{(G)} \qquad 1 < p < \infty \qquad \text{if } n = 1, 2; \qquad 1 < p < \frac{n+2}{n-2} \qquad \text{if } n \geq 3.$$

In [4], Xu and Su considered the well-posedness of problem (0.1) when the initial data are at different energy levels $J(u_0) < d$, $J(u_0) = d$, $J(u_0) > d$ respectively. Especially, they obtained the blow-up phenomena of solutions with low initial energy $J(u_0) < d$ ([4, Theorem 11]) and critical energy $J(u_0) = d$ ([4, Theorem 14]), respectively. However, there are some bugs in the proofs of the blow-up phenomena of solution, due to the confused definition of $\|u\|^2_{H_0^1}$. While proving Theorem 11 and Theorem 14, the authors defined in [4] at first $\|u\|^2_{H_0^1}$ as $\|u\|_2^2 + \|\nabla u\|_2^2$ in line 10 on page 2745, but replaced $\|\nabla u\|_2^2$ with $\|u\|^2_{H_0^1}$ directly in lines 14 and 15 on page 2745 and lines 2–3 on page 2746, and replaced $\|u\|^2_{H_0^1}$ with $\|\nabla u\|_2^2$ in lines 8 and 10 on page 2746. This confusion exists also on pages 2734–2745 before Theorem 11 of [4]. In this short note, we shall amend the proofs of [4, Theorem 11] and [4, Theorem 14] by rewriting these two proofs, respectively.

Condition (H) in [4] is replaced by (G) in [5] as a correction in order to satisfy the Palais–Smale condition, to ensure compact and to make sure that problem (0.1) with any geometry of the domain exists the smooth nontrivial stationary solutions. Throughout this note, all other assumptions are the same as [4].

We recall the following functionals

$$J(u) = \frac{1}{2}\|\nabla u\|_2^2 - \frac{1}{p+1}\|u\|_{p+1}^{p+1},$$
$$I(u) = \|\nabla u\|_2^2 - \|u\|_{p+1}^{p+1},$$

the Nehari manifold

$$\mathcal{N} = \{u \in H_0^1(\Omega) \mid I(u) = 0,\ \|\nabla u\|_2^2 \neq 0\},$$

and the mountain pass level

$$d = \inf_{u \in \mathcal{N}} J(u). \tag{0.2}$$

We adopt the fixed definition of $H_0^1(\Omega)$-norm as

$$\|u\|^2_{H_0^1} = \|u\|_2^2 + \|\nabla u\|_2^2, \qquad \forall\ u \in H_0^1(\Omega). \tag{0.3}$$

To avoid the confusion, we remark that on pages 2734–2745 before [4, Theorem 11] all $\|u\|_{H_0^1}$ ($\|u_m\|_{H_0^1}$) should be changed to $\|\nabla u\|_2$ ($\|\nabla u_m\|_2$), except for $\int_0^t \|u_\tau\|^2_{H_0^1} \mathrm{d}\tau$, $\int_0^T \|u\|^2_{H_0^1} \mathrm{d}\tau$ and $\int_0^t \|u_{m\tau}\|^2_{H_0^1} \mathrm{d}\tau$.

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