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THE BREZIS-NIRENBERG PROBLEM FOR THE CURL-CURL OPERATOR

JAROSŁAW MEDERSKI

ABSTRACT. We look for solutions $E : \Omega \rightarrow \mathbb{R}^3$ of the problem

$$\begin{cases} \nabla \times (\nabla \times E) + \lambda E = |E|^{p-2}E & \text{in } \Omega \\ \nu \times E = 0 & \text{on } \partial\Omega \end{cases}$$

on a bounded Lipschitz domain $\Omega \subset \mathbb{R}^3$, where $\nabla \times$ denotes the curl operator in \mathbb{R}^3 . The equation describes the propagation of the time-harmonic electric field $\Re\{E(x)e^{i\omega t}\}$ in a nonlinear isotropic material Ω with $\lambda = -\mu\varepsilon\omega^2 \leq 0$, where μ and ε stand for the permeability and the linear part of the permittivity of the material. The nonlinear term $|E|^{p-2}E$ with $p > 2$ is responsible for the nonlinear polarisation of Ω and the boundary conditions are those for Ω surrounded by a perfect conductor. The problem has a variational structure and we deal with the critical value p , for instance, in convex domains Ω or in domains with $\mathcal{C}^{1,1}$ boundary, $p = 6 = 2^*$ is the Sobolev critical exponent and we get the quintic nonlinearity in the equation. We show that there exist a cylindrically symmetric ground state solution and a finite number of cylindrically symmetric bound states depending on $\lambda \leq 0$. We develop a new critical point theory which allows to solve the problem, and which enables us to treat more general anisotropic media as well as other variational problems.

MSC 2010: Primary: 35Q60; Secondary: 35J20, 58E05, 35B33, 78A25

Key words: time-harmonic Maxwell equations, perfect conductor, ground state, variational methods, strongly indefinite functional, Nehari-Pankov manifold, Brezis-Nirenberg problem, critical exponent.

1. INTRODUCTION

The following equation

$$\nabla \times (\mu^{-1} \nabla \times \mathcal{E}) + \varepsilon \partial_t^2 \mathcal{E} = -\partial_t^2 \mathcal{P}_{NL}.$$

describes the propagation of the electric field \mathcal{E} in a nonlinear bounded medium Ω with the permeability μ , the linear part of the permittivity ε and the nonlinear polarisation \mathcal{P}_{NL} ; see Saleh and Teich [34]. In the time-harmonic case the fields \mathcal{E} and \mathcal{P}_{NL} are of the form $\mathcal{E}(x, t) =$

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