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# DILATIONS OF OPERATOR-VALUED MEASURES WITH BOUNDED $p$-VARIATIONS AND FRAMINGS ON BANACH SPACES

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Abstract. The dilations for operator-valued measures (OVMs) and bounded linear maps indicate that the dilation theory is in general heavily dependent on the Banach space nature of the dilation spaces. This naturally led to many questions concerning special type of dilations. In particular it is not known whether ultraweakly continuous (normal) maps can be dilated to ultraweakly continuous homomorphisms. We answer this question affirmatively for the case when the domain algebra is an abelian von Neumann algebra. It is well known that completely bounded Hilbert space operator valued measures correspond to the existence of orthogonal projection-valued dilations in the sense of Naimark and Stinespring, and OVMs with bounded total variations are completely bounded but not the vice-versa. With the aim of classifying OVMs from the dilation point of view, we introduce the concept of total $p$-variations for OVMs. We prove that any completely bounded OVM has finite 2-variation, and any OVM with finite $p$-variation can be dilated to a (but usually non-Hilbertian) projection-valued measure of the same type. With the help of framing induced OVMs, we prove that conventional minimal dilation space of a non-trivial framing contains $c_0$, then does not have bounded $p$-variation.

Contents



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