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Spectral gap on Riemannian path space over static and evolving manifolds

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A B S T R A C T

In this article, we continue the discussion of Fang–Wu (2015) to estimate the spectral gap of the Ornstein–Uhlenbeck operator on path space over a Riemannian manifold of pinched Ricci curvature. Along with explicit estimates we study the short-time asymptotics of the spectral gap. The results are then extended to the path space of Riemannian manifolds evolving under a geometric flow. Our paper is strongly motivated by Naber's recent work (2015) on characterizing bounded Ricci curvature through stochastic analysis on path space.

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1. Introduction

Let (M, g) be a d-dimensional complete smooth Riemannian manifold with ∇ and Δ denoting respectively the Levi-Civita connection and the Laplacian on M. Given a C^1 vector field Z on M, we consider the Bakry–Emery curvature

$$\mathrm{Ric}^Z := \mathrm{Ric} - \nabla Z$$

for the so-called Witten Laplacian $L = \Delta + Z$ where Ric is the Ricci curvature tensor with respect to g. It is well known that the spectral gap of L can be estimated in terms of a lower curvature bound K, i.e.,

$$\mathrm{Ric}^Z \geq K$$

for some constant K, see e.g. [5,6,10]. These results reveal the close relationship between spectral gap, convergence to equilibrium and hypercontractivity of the corresponding semigroup. For example, Poincaré inequalities and log-Sobolev inequalities which can be used to characterize the convergence for the semigroup, imply certain lower bound for the spectral gap.

In this article, we extend this circle of ideas to the Riemannian path space over M and revisit the problem of estimating the spectral gap of the Ornstein–Uhlenbeck operator under the following general curvature condition: there exist constants k_1 and k_2 such that

$$k_1 \leq \mathrm{Ric}^Z \leq k_2.$$

Before moving on, let us briefly summarize some background results on stochastic analysis on path space over a Riemannian manifold. Stochastic analysis on path space attracted a lot of attention since 1992 when B.K. Driver proved quasi-invariance of the Wiener measure on the path space over a compact Riemannian manifold [11]. A milestone in the theory is the integration by parts formula (see e.g. [3,15]) for the associated gradient operator induced by the quasi-invariant flow. This result is a main tool in proving functional inequalities for the corresponding Dirichlet form, for instance, the log-Sobolev inequality [1]; the constant in this inequality has been estimated in [19] in terms of curvature bounds.

Very recently, A. Naber [24] proved that certain log-Sobolev inequalities and L^p-inequalities on path space are equivalent to an upper bound for the norm of Ricci curvature on the base manifold M; R. Haslhofer and A. Naber [17] extended these results to characterize solutions of the Ricci flow, see also [18]. Inspired by this work, S. Fang and B. Wu [16] gave an estimate of the spectral gap under the curvature condition that

$$k_1 \leq \mathrm{Ric}^Z \leq k_2$$