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Limit theory for random walks in degenerate time-dependent random environments

Marek Biskup [a,b,*], Pierre-François Rodriguez [a]

[a] *Department of Mathematics, UCLA, Los Angeles, California, USA*
[b] *Center for Theoretical Study, Charles University, Prague, Czech Republic*

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ABSTRACT

We study continuous-time (variable speed) random walks in random environments on $\mathbb{Z}^d$, $d \geq 2$, where, at time t, the walk at x jumps across edge (x, y) at time-dependent rate $a_t(x, y)$. The rates, which we assume stationary and ergodic with respect to space–time shifts, are symmetric and bounded but possibly degenerate in the sense that the total jump rate from a vertex may vanish over finite intervals of time. We formulate conditions on the environment under which the law of diffusively-scaled random walk path tends to Brownian motion for almost every sample of the rates. The proofs invoke Moser iteration to prove sublinearity of the corrector in pointwise sense; a key additional input is a conversion of certain weighted energy norms to ordinary ones. Our conclusions apply to random walks on dynamical bond percolation and interacting particle systems as well as to random walks arising from the Helffer–Sjöstrand representation of gradient models with certain non-strictly convex potentials.

* Corresponding author.
E-mail addresses: biskup@math.ucla.edu (M. Biskup), rodriguez@math.ucla.edu (P.-F. Rodriguez).

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1. Introduction

1.1. Model and assumptions

The aim of this note is to study the long-time behavior of random walks on $\mathbb{Z}^d$, $d \geq 2$, in a class of dynamical random environments given as a family of non-negative random variables

$$\big\{a_t(e)\colon e \in E(\mathbb{Z}^d),\, t \in \mathbb{R}\big\}, \tag{1.1}$$

where $E(\mathbb{Z}^d)$ denotes the set of (unordered) nearest-neighbor edges of $\mathbb{Z}^d$. For each sample of these random variables, referred to as conductances, we consider the continuous time Markov chain $\{X_t\colon t \geq 0\}$ on $\mathbb{Z}^d$ with the instantaneous generator L_t acting on functions $f\colon \mathbb{Z}^d \to \mathbb{R}$ as

$$L_t f(x) := \sum_{y\colon\, |y-x|=1} a_t(x,y)\big[f(y) - f(x)\big]. \tag{1.2}$$

The variable $a_t(e) = a_t(x,y)$, i.e., the jump rate of the walk across edge $e = (x,y)$ at time t, is assumed to obey $a_t(e) \in [0,1]$ with $a_t(e) = 0$ allowed for non-trivial finite intervals of time. Our aim is to describe the long-time behavior of such random walks and, in particular, to show that their path distribution, scaled diffusively, tends to a non-degenerate Brownian motion.

A representative example of the above setting is the variable-speed random walk on dynamical bond percolation on $\mathbb{Z}^d$. In this case $a_t(e)$ is, for each $e \in E(\mathbb{Z}^d)$, an independent copy of a stationary continuous-time process on $\{0,1\}$ with joint invariant distribution (product) Bernoulli(p) for some prescribed $p \in (0,1)$. We interpret $a_t(e) = 1$ as the event that edge e is occupied at time t and $a_t(e) = 0$ as the event that edge e is vacant. The random walk then jumps at rate 1 across edges incident with its current position that are occupied at that instant of time. When the site where the walk is located has no incident occupied edges, the walk does not move.

It is clear that some mixing properties of the conductances (1.1) in both space and time are required for the desired convergence to Brownian motion to be possible. We will work under the following set of technical assumptions:

Assumption 1.1. The family $\{a_t(e)\colon e \in E(\mathbb{Z}^d),\, t \in \mathbb{R}\}$ is realized as coordinate projections on the product space $\Omega := [0,\infty)^{\mathbb{R}\times E(\mathbb{Z}^d)}$ endowed with the product Borel σ-algebra $\mathcal{F}$ and the probability distribution denoted by $\mathbb{P}$. In addition, we assume:

(1) $t \mapsto a_t(e)$ obeys

$$a_t(e) \in [0,1] \tag{1.3}$$

for each $e \in E(\mathbb{Z}^d)$ and each $t \in \mathbb{R}$,