# Accepted Manuscript


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PII: S0022-1236(17)30374-9
DOI: https://doi.org/10.1016/j.jfa.2017.09.014
Reference: YJFAN 7885

To appear in: *Journal of Functional Analysis*

Received date: 11 April 2017
Accepted date: 20 September 2017

Please cite this article in press as: V. Fischer et al., Nilpotent Gelfand pairs and Schwartz extensions of spherical transforms via quotient pairs, *J. Funct. Anal.* (2017), https://doi.org/10.1016/j.jfa.2017.09.014

# NILPOTENT GELFAND PAIRS AND SCHWARTZ EXTENSIONS OF SPHERICAL TRANSFORMS VIA QUOTIENT PAIRS

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Abstract. It has been shown [1, 2, 9, 10] that for several nilpotent Gelfand pairs $(N, K)$ (i.e., with $N$ a nilpotent Lie group, $K$ a compact group of automorphisms of $N$ and the algebra $L^1(N)^K$ commutative) the spherical transform establishes a 1-to-1 correspondence between the space $\mathcal{S}(N)^K$ of $K$-invariant Schwartz functions on $N$ and the space $\mathcal{S}(\Sigma)$ of functions on the Gelfand spectrum $\Sigma$ of $L^1(N)^K$ which extend to Schwartz functions on $\mathbb{R}^d$, once $\Sigma$ is suitably embedded in $\mathbb{R}^d$. We call this *property (S)*.

We present here a general bootstrapping method which allows to establish property (S) to new nilpotent pairs $(N, K)$, once the same property is known for a class of *quotient pairs* of $(N, K)$ and a $K$-invariant form of Hadamard formula holds on $N$.

We finally show how our method can be recursively applied to prove property (S) for a significant class of nilpotent Gelfand pairs.

Contents



2010 *Mathematics Subject Classification.* Primary: 43A22; Secondary: 13A50, 43A85, 43A90, 43A20.

*Key words and phrases.* Fourier and spectral multipliers, analysis on nilpotent Lie groups, Gelfand pairs and spherical transform, Invariants.