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Sobolev multipliers, maximal functions and parabolic equations with a quadratic nonlinearity

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ABSTRACT

We develop a general framework to describe global mild solutions to a Cauchy problem with small initial values concerning a general class of semilinear parabolic equations with a quadratic nonlinearity. This class includes the Navier–Stokes equations, the subcritical dissipative quasi-geostrophic equation and the parabolic–elliptic Keller–Segel system.

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1. Presentation of the results

In this paper, we shall study parabolic semi-linear equations on $(0, +\infty) \times \mathbb{R}^n$ of the type:

$$\partial_t u + (-\Delta)^{\alpha/2} u = (-\Delta)^{\beta/2} u^2 \tag{1}$$

with $0 < \alpha < n + 2\beta$ and $0 < \beta < \alpha$.

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More generally, we consider the following Cauchy problem: given $\vec{u}_0 \in (\mathcal{S}'(\mathbb{R}^n))^d$, find a vector distribution $\vec{u}$ on $(0, +\infty) \times \mathbb{R}^n$ (or on $(0, T) \times \mathbb{R}^n$) such that, for $i = 1, \ldots, d$, we have

$$\partial_t u_i = -(-\Delta)^{\alpha/2} u_i + \sum_{j=1}^{d} \sum_{k=1}^{d} \sigma_{i,j,k}(D)(u_j u_k) \tag{2}$$

and

$$\lim_{t \to 0} u_i(t, x) = u_{i,0}. \tag{3}$$

We assume that $\sigma_{i,j,k}(D)$ is a homogeneous pseudo-differential operator of degree β with $0 < \beta < \alpha < n + 2\beta$: for $f \in \mathcal{S}(\mathbb{R}^n)$ with Fourier transform $\mathcal{F}f$, we have:

$$\sigma_{i,j,k}(D) f = \mathcal{F}^{-1}\big(\sigma_{i,j,k}(\xi) \mathcal{F}f(\xi)\big) \tag{4}$$

where $\sigma_{i,j,k}$ is a smooth (positively) homogeneous function of degree β on $\mathbb{R}^n - \{0\}$:

$$\text{for } \lambda > 0 \text{ and } \xi \neq 0, \quad \sigma_{i,j,k}(\lambda\xi) = \lambda^\beta \sigma_{i,j,k}(\xi). \tag{5}$$

We rewrite equation (2) in a vectorial form:

$$\partial_t \vec{u} = -(-\Delta)^{\alpha/2} \vec{u} + \sigma(D)(\vec{u} \otimes \vec{u}) \tag{6}$$

and use Duhamel's formula to transform the problem into an integral problem:

$$\vec{u} = e^{-t(-\Delta)^{\alpha/2}} \vec{u}_0 + \int_0^t e^{-(t-s)(-\Delta)^{\alpha/2}} \sigma(D)(\vec{u} \otimes \vec{u}) \, ds. \tag{7}$$

We shall use the classical estimate:

Lemma 1. *There exists a constant C_0 (depending on σ) such that, for two functions $\vec{u}$ and $\vec{v}$ on $\mathbb{R}^n$ (with values in $\mathbb{R}^d$) we have*

$$|e^{-(t-s)(-\Delta)^{\alpha/2}} \sigma(D)(\vec{u} \otimes \vec{v})| \leq C_0 \int_{\mathbb{R}^n} \frac{|\vec{u}(y)||\vec{v}(y)|}{(|t-s|^{1/\alpha} + |x-y|)^{n+\beta}} \, dy. \tag{8}$$

Due to homogeneity, this lemma is a direct consequence of Lemma 7 which will be proved in Appendix A.

The core of the paper is the discussion of the equation

$$U(t, x) = U_0(t, x) + C_0 \iint_{\mathbb{R} \times \mathbb{R}^n} K_{\alpha,\beta}(t-s, x-y) \, U^2(s, y) \, ds \, dy \tag{9}$$

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