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Partial Hölder regularity for elliptic systems with non-standard growth

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We study partial Hölder regularity for elliptic systems with non-standard growth. We consider general systems with Orlicz growth and discontinuous coefficient factor, for which we prove that their weak solutions are partially Hölder continuous for any Hölder exponents. In addition, we also obtain a similar result for systems with double phase growth.

1. Introduction

In this paper, we study the partial Hölder continuity for elliptic systems with non-standard growth. Partial regularity of weak solutions to elliptic systems with p-growth, which is usually called 'standard growth', has been extensively studied since the basic works of Campanato [6,7] after the pioneering works of Giusti & Miranda and Morrey [20,38]. In particular, the main question was that with respect to conditions of the coef-

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ficient factor we can obtain partial regularity which are the natural counter parts of ones for single equations. For Hölder continuous coefficients, it has been well known that the gradient of weak solutions are Hölder continuous except a Lebesgue measure zero set. We refer to for instance a overview paper [37] of Mingione, in which one can find a lot of references and an historical account of the problem. On the other hand, the problem of establishing partial regularity results in the case coefficients are only continuous has remained open for a long time. In this situation, in analogy to the scalar case, the type of partial regularity expected is the Hölder continuity of solutions with any Hölder exponent outside a Lebesgue measure zero set, called indeed the singular set; no pointwise regularity of the gradient is expected even in the scalar case. After various attempts, this conjecture has been finally proved by Foss & Mingione [17] when $p \geq 2$; in this paper the authors treated both the case of elliptic systems and the one of minimizers of quasi-convex functionals. We also refer to [27–29] for partial regularity in the functional setting and related estimates on singular sets Hausdorff dimension. We point out that no singular set dimension estimate is known in this case.

Problems with non-standard growth conditions were first systematically studied by Marcellini in series of fundamental papers [33–36]. In addition, various non-standard growth conditions have been presented. Amongst them, let us introduce three significant non-standard growth conditions. The first one is the so-called Orlicz growth condition which is related to the so-called Orlicz function $G(t)$ described below. Orlicz spaces have been studied for a long time, and Lieberman first applied this to partial differential equations and functionals in [30–32]. We refer to for instance [2,9,14] and references therein for regularity results about problems with Orlicz growth. The second one is the $p(x)$-growth condition which is related to the function $t \mapsto t^{p(x)}$. Problems with $p(x)$-growth are motivated by modeling of electrorheological fluids [42] and image restoration [8], and so, for last two decades, there have been intensive research activities about problems with $p(x)$-growth, see [1,5,13,23] for regularity results and related references. The final one, that has recently attracted a lot of attention, is the so-called double phase growth condition which is related to the function $H(x,t)$ described below. Double phase problems are motivated by the modeling of the mixtures of two different materials in the context of Homogenization theory [44] and one of typical examples of Lavrentiev's phenomenon [43]. Recently, Baroni & Colombo & Mingione have obtained various regularity results for these problems [3,4,10–12]. We also refer to [18,39] for a borderline growth condition related to the function $t^{p(x)}\log(e+t)$, and [22,24,25] for the so-called generalized Orlicz growth condition.

The non-standard growth conditions we consider in this paper are the Orlicz growth condition and the double phase condition. Let us first introduce our result for systems with Orlicz growth. Suppose that $G:[0,\infty)\to[0,\infty)$ with $G(0)=0$ is C^2 and satisfies

$$1 < g_1 - 1 \leq \inf_{t>0}\frac{tG''(t)}{G'(t)} \leq \sup_{t>0}\frac{tG''(t)}{G'(t)} \leq g_2 - 1 \tag{1.1}$$

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